Impact of Hedge Funds on Traditional Investment Products

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The purpose of this paper is to present the hedge fund industry in order to demonstrate that their primary interest in asset management is to diversify a standard portfolio because of their decorrelation with markets, and this by conducting research on two areas: First of all we will analyze the correlation that may exist between alternative and traditional investment products. Secondly we generate portfolio diversification thanks to the dynamic investment model in discrete time combined with the approach for evaluating the empirical probability. Our results emphasize the importance of the criterion of belonging to an investment strategy for a hedge fund.

INTRODUCTION

Hedge funds are now one of the most popular in financial circles and in anything related to the economy in general. Bear markets are not strangers to this, because hedge funds have absolute return as target, that is to say that managers of such funds seek to provide a positive return, regardless of changes in the market as a whole. To achieve this, managers implement investment strategies different to the classic one. Most of them specialize in very specific niches such as convertible bonds, distressed companies or restructuring, macroeconomic movements or global emerging markets and they often use derivative products, they are usually paid on the basis of commission related to the performance.

One feature of most hedge funds highlighted is their low correlation with traditional investment products such as stocks, bonds and raw materials. Several studies have been made to clarify this relationship, such as Ackermann et al. (1999), Agarwal and Naik (1999), Liang (1999), Edwards and Caglayan (2000), Amin and Kat (2001), Capocci and Hübner (2003) who put this forward. Except that their work is often limited to analyze the correlation between hedge funds, stocks and bonds, without considering other investment products such as corporate bonds, commodities or stocks, it's actually the first point deals with the study, in fact, we will analyze the correlation between hedge funds (taken together, by investment strategy and individually), actions, bonds and raw materials to determine the level of correlation, its evolution and its stability time.

The low correlation would be one of the key benefits of hedge funds because if one refers to the modern portfolio theory of Markowitz (1952), the low correlation of an asset class with traditional media investment allows investors to achieve efficient frontier is more interesting to say that the investor gets a better return for the same risk or lower risk for a similar yield and this allows to build a diversified portfolio.

To study the correlation recent research has shown that the characteristics of hedge fund returns are more complex than those of stocks and bonds. Thus studies have been conducted to propose a methodology that represents an alternative to the standard analysis. Indeed, this methodology constructs

and rebalances portfolios composed of stocks, bonds and commodities. Grauer and Hakansson (1995) applied the dynamic investment model in discrete time to show the gain diversification through Real Estate (real estate). Thus, these authors were able to demonstrate a significant gain due to diversification through real estate.

However, one question remains, no study has confirmed or denied: the properties of diversification offered by hedge fund strategies are they different from traditional assets? ... So the second purpose of this paper is to study the difference between diversification offered through a general index of hedge funds relative to diversification through a set of indices of hedge fund strategies.

This article represents an extension of the literature on the correlation and diversification across hedge fund strategies. This study is evaluating the gain of diversification across hedge funds over a number of traditional assets using the dynamic investment model in discrete time of Grauer and Hakansson (1985, 1986, 1987, 1995, 2001) combined with the empirical probability assessment approach (EPAA). Indeed in this article there will be a further analysis of Hagelin and Promberger (2003) by examining the returns of a portfolio of different classes of shares and indices of hedge fund strategies.

This paper is structured as follows: after a review of the literature, we conduct an analysis by grouping funds by investment strategy in composite indices and recitals individually. We also undertake an analysis of different sub-periods to check the consistency of previous results, then we will present data and the assumptions and methodology that is the model of discrete-time dynamic investment approach combined with evaluation of the empirical probability, the third part presents the results obtained from evaluating the performance of diversified portfolios containing equity indices, bonds and commodities with and without hedge funds to finally conclude.

LITERATURE REVIEW

Hedge funds are now the subject of numerous studies. It seems interesting to make an inventory of the main studies. Ackermann and Ravenscraft (1998) highlight the fact that the more stringent restrictions on mutual funds than for hedge funds affect their performance. Ackermann et al (1999) and Liang (2001), who compared the performance of hedge funds than mutual funds and indices, indicated that hedge funds have consistently achieved better performance than mutual funds but was not better than the indices used. Amenc and Martellini (2002) have shown that the inclusion of hedge funds in a portfolio can significantly improve the risk-return trade-off by their very nature limited correlation with other securities. This low correlation was also highlighted by Liang (1999) and Agarwal and Naik (2000a). According to Brown et al. (2001), hedge funds that offer good performance in the first part of the year reduce the volatility in the second half of the year. They provide the annual performance of sorts. Capocci and Hübner (2003) show the ability of some hedge funds to outperform the market over time.

Point seen diversification of academic research has focused on the sources of hedge fund returns. Other studies have been done on the performance of hedge funds without taking into account the differentiation factors, styles and characteristics associated with abnormal returns of these funds. Brown, Goetzmann and Ibbotson (1999) studied a sample of offshore hedge funds between 1989 and 1995, and found a positive risk-adjusted. Their results did not support the hypothesis of the effect of the difference of talent managers, nor the persistence of the performance of some managers.

Unlike Brown, Goetzmann and Ibbotson (1999), Ackermann, Mc Enally and Ravenscraft (1999) observed that, despite the fact that hedge funds have outperformed mutual funds in the period 1988-1995, they have not performed on the index of market standard in terms of yield risk over the same period, this is mostly due to poor performance of hedge funds in 1994 and 1995. They studied the commissions offered to managers, as a percentage of the gain beyond a break-even ("High Water Marks") and found a significant relationship between commissions and profitability.

In their combined sample from the two databases, Ackermann, Mc Enally and Ravenscraft (1999) have included data on funds that have disappeared and whose omission could bias the study to eliminate survivorship bias "Survivorship bias ". Ackermann, Mc Enally and Ravenscraft (1999) support the idea that survivorship bias erroneous results as suggested by other studies. Indeed, they believe that another

source of bias against balance survivorship bias caused by the hedge funds that perform well and which fail to disclose their results. This self-selection bias (self-selection bias) may be due to the desire of certain funds to remain discreet and maintain confidentiality of their strategy and their yields. Although the sample of Ackermann, Mc Enally and Ravenscraft (1999) is larger than that of Brown, Goetzmann and Ibbotson (1999), they did not take into account the data for the period before 1999 that experienced the Asian crisis in 1997-1998 and the collapse of the LTCM fund in 1998 was highly publicized. Their studies have focused on several questions about different periods and with a larger sample, and their results contradict most existing studies replies that hedge funds outperform traditional benchmarks.

On the other hand, Brown, and Goetzma Ibbotson (1999) and Agarwal and Naik (1999) reach opposite conclusions with respect to the issue of performance persistence of hedge funds. Indeed, Brown, Goetzmann and Ibbotson (1999) observed a lack of persistence in performance, while Agarwal and Naik (1999) have shown the existence of a form of persistence of performance using data from the database HFR (Hedge Fund Research), they concluded that the persistence is less observed when using a multifactor model than when using a two-period model. They also showed that more the range of returns increases, the most the of persistence decreases. They attributed the persistence observed in the intervals and in the two-period model, not the losers that continue to be losers, only winners who continue to be winners.

Hagelin and Promberger (2003) have taken the model used by Grauer and Hakansson (1985, 1986, 1987, 1995and 2001) to study a portfolio of stocks and bonds with and without hedge funds. For this they used as a proxy for hedge fund index "HFR Fund Weighted Composite index" of the database Hedge Fund Research (HFR), then an index fund of funds (HFR fund of funds index) of same basis. These authors showed an increase in the average profitability of the portfolio following the addition of the index HFR index without increasing the variance of the portfolio. Unlike the first result, diversification through an index fund of funds does not show a gain in terms of profitability, variance, the authors explain this result across the board double as support funds of funds and the problem of bias faced by hedge fund data.

Harry M. Kat (2005) in her article, shows how investors can neutralize the unwanted skewness and kurtosis effects from investing in hedge funds by 1) purchasing out-of-the-money equity puts, 2) investing in managed futures, and/or by 3) overweighting equity market neutral and global macro and avoiding distressed securities and emerging market funds. The analysis suggests that all three alternatives are up to the job but also come with their own specific price tag.

Bacmann, Jeanneret and Scholz (2008) measure the relationship between two time series from a static perspective. They introduce a methodology to measure the drivers of correlations between the returns of a dynamic strategy (hedge funds) and those of a static benchmark (traditional investments). Their approach explains why correlations between hedge funds and traditional investments are sometimes high and at other times low. Furthermore they highlight which elements of hedge funds strategies are responsible for making correlations vary over time. It constitutes an improvement in measuring the diversification potential of hedge funds to traditional investments.

Mikael Haglund (2010), in her article, uses higher moment betas to examine the effects on portfolio volatility, skewness and kurtosis when hedge funds are added to an equity portfolio. The results show that hedge funds, in general, can lower the volatility, skewness and kurtosis of the portfolio but large variations are seen between different hedge funds strategies. Convertible Arbitrage, Equity Market Neutral, Fixed Income Arbitrage, Merger Arbitrage and Macro are identified as the most attractive strategies to include in an equity portfolio for investors who care about higher moment risks and want to limit downside risk.

In this study, we deepen the analysis of Bacmann, Jeanneret and Scholz (2008) and Promberger and Hagelin (2003) by examining the returns of a portfolio of different classes of shares and indices of hedge fund strategies. Our contribution to the literature is twofold: First, we wish to highlight the existence of a gain diversification through a general index, but also through a set of indices of hedge fund strategies, in a second time, we evaluate the contribution of each hedge fund strategies portfolio of traditional assets. Three types of portfolios are studied: a portfolio of traditional assets (stocks and an index of real estate), a portfolio of traditional assets and hedge funds a general index and a portfolio of

traditional assets and indices of hedge fund strategies. The data used in their articles are retrieved from the database "Hedge indexes", these data have the advantage of allowing to overcome the survivorship bias (Survivorship bias) since they included data on age or missing in funds during the study period.

HEDGE FUNDS: PERFORMANCE UNCORRELATED SOURCE FOR INVESTORS AND DIVERSIFICATION TOOL

Data and Descriptive Statistics

Prior to the analysis of correlations, we proceed to the descriptive analysis of our database.

In our study, we use data on the strategies from the database "Hedge indexes". We have net monthly returns of individual funds. The data cover the period from January 1994 to December 2010.

We also have monthly returns of various equity indices, bonds and commodities. These indices are:

Indices actions :	**Indices obligations :**	**Indices matières premières :**
✓ Le S&P 500	✓ JPM chase L	✓Goldman Sachs Commodity
✓ Le MSCI	✓ Le LH Lehman	

The first table in appendix 1 reports the descriptive statistics of strategies[1] and indices.

The table shows that the best monthly performance returns to the LES strategy with a monthly average performance of 1.1% followed by D EV strategy with 1.09% then the GM strategy with 1.03%. Strategies and sub strategies that have the worst monthly returns are DSB with -0.04%, MF with 0.44% and FIA with a monthly average performance of 0.51%.

When we consider also the risk according to the Sharpe ratio, the strategies which are distinguished are the EMN strategy with a monthly ratio of 2.161291, followed by RA EV strategy with 1.529584 and FIA strategy with 1.475149. The strategy MA Sharpe ratio lowest i.e. 0.02. Taking into account the t-statistics, the majority of monthly returns are significantly different from zero at the 5%, except for the strategy MF and DSB.

The second part of the table reports the descriptive statistics of the indices. It indicates that the highest average performance is offered by the LH index. The other strategies are getting the lowest performance throughout the period.

The t-statistics indicate that all indices (except JPM and GD) provide returns which are significantly different from zero at the 1%.

Analysis of Correlations Between Strategies and Indices

We begin the analysis by studying the correlation between hedge funds (grouped by strategy) and the benchmarks used. Subsequently, we analyze the correlations between hedge funds; we will complete the study with an analysis of correlations between hedge funds and conventional indices over a rolling period.

To calculate the correlation, we rely on a traditional measure which is the correlation coefficient. Recall that the value of the correlation coefficients varies between -1 and +1, and when the coefficient is near to 1, the underlying media follows the same trend, this coefficient is given by the following formula:

$$\rho = \frac{\sigma_{(p,q)}}{\sigma_p \times \sigma_q}$$

with ρ: correlation coefficient between p and q
$\sigma_{(p, q)}$: covariance between p and q
σ_p and σ_q : respectively the standard deviation of p and q

Correlation Between Hedge Funds Pooled by Strategies and Indices

The results of the analysis of correlations between hedge funds (grouped by investment strategy) and the market indices used are shown in the table 2 of the first appendix:

The idea behind this first analysis is to know if each strategy can be significantly correlated with one or more indices. For this, we gather in the same matrix, the correlation of different strategies and sub-

strategies. First we analyze the figures in table horizontally (as in strategy and strategy) and then vertically (relation between indices and strategies).

Strategies CA, EM, EMN, EV, DEV, MS EV, RA EV, GM, and MS are all significantly positively correlated with all of the equity indices. Moreover, they are not significantly correlated with the index LH. This shows that these strategies are essentially based on equities, products linked equities or high yield bonds. This is easily explained by event strategies investing in high yield bonds and stocks, but it is surprising that macro funds are not more strongly correlated to the bond market. In addition, these funds are generally diversified in some market and invest in different products, which justifies that all correlation coefficients are between -0.8 and 0.6.

The strategy LES is significantly correlated with the equity indices at the 1%, reflecting the fact that managers combine long and short positions in equities generally without recourse to non-linear derivative products.

The GM strategy is significantly correlated with all equity indices. This is due to its international aspect generally invested in other continents as North America. This strategy is also weakly correlated with the bond indices.

The general indices CS was significantly and positively correlated with all indices except for the classic LH index where it is negatively correlated.

The strategy DSB is significantly correlated with the LH index. EMN strategy is positively correlated with all indices.

If we analyze the table vertically, we find that the correlations are very different from one strategy to another. If we do not consider the strategy DSB highly decorrelated, the strategies are all positively correlated with all indices (with some exceptions). There are minimal negative correlations especially in the case of the LH index.

For the bond market, the correlation coefficients also vary greatly. They ranged from 0.440 for the strategy EV significant at 5% to -0.4203 but not significant for the strategy DSB.

For the commodities market they range from -0.0766 to MS EV strategy, to -0.077 for the strategy DSB.

The three strategies in D EV, MS EV and RA EV are correlated with the same ratings as the mother strategy.

In this first analysis, we conclude that there are two types of strategies, those that can be grouped into families and others cannot be combined. The First Family (LES, RA EV, EV MS, D EV, EV, EM and CA) includes strategies all significantly related to equity indices, JPM, and GD. The second family (FIA, MF and MS) includes strategies significantly uncorrelated with all indices. The strategy DSB is itself due to its low correlation with all markets used. Other strategies are ultimately to be significantly correlated with virtually all indices (MNEs) or correlated with some indices close to the investment products they prefer.

Correlation Between the Strategies of Hedge Funds

Table 3 in appendix 1 reports the correlation coefficients of alternative strategies. We notice a significant difference between the observed coefficients, ranging from a coefficient of 0.741 (significantly different from zero at the 1% level) between strategies DEV and EV, to a coefficient of -0.541 (significantly negative at the 1%) between the strategies LES and DSB. Note that the strategy DSB is negatively correlated with all other strategies.

Correlation Analysis Strategies with Clues Rolling Period

Before concluding the analysis of correlation, it seems interesting to conduct a final analysis. We will examine the correlations of hedge fund strategies with benchmarks on various sub-periods in order to determine whether there is constant correlation coefficient over time. Then we will focus on various periods of crisis before determining if a pattern emerges from numbers.

From our global analysis period of 192 months (January 1994-December 2010), it emerges eight rolling periods which are:

- January 1994 to December 1995
- January 1996 to December 1997
- January 1998 to December 1999
- January 2000 to December 2001
- January 2002 to December 2003
- January 2004 to December 2005
- January 2006 to December 2007
- January 2008 to December 2010

Based on numbers in appendix 2 we constructed eight correlation matrices (one per period).

These eight correlation matrices report specific outcomes by strategy and classic. They indicate that globally:

✓ The correlation between a strategy and hedge fund and an index can vary significantly over time (ex the correlation between CA and the SP index decreased from 0.398, significantly different from zero on the first sub-period to 0.094 significant in the second before increasing during periods the other periods and finally be significantly different from zero);

✓ The correlation of all indices with strategies typically progresses in the same direction over the different sub-periods analyzed (ex increase in overall correlations of the first to the second sub-period);

✓ Correlations are less pronounced on the first sub-period as they are on the other seven periods analyzed;

Note that the FIA strategy is not always significantly correlated with classic indices over the eight sub-periods, indicating a source of diversification for investors. The other strategies are significantly correlated with equity markets and then only in some cases.

Let's look to economic events occurring during these periods to see if we can make a comparison between the rolling periods where the coefficients are highest, and periods of particular market.

Among the economic events during the analysis, we note the increase in American interest rates that occurred between February and April 1994.

The performance of each strategies and bonds are summarized in the table 4 of appendex1.

There is also the Asian crisis between August and October 1997, occurring during the second period, where, again, the coefficients were highest. The Russian crisis (July-September 1998) involved in the third period of analysis. During this period, the correlation between the indices and most of the hedge fund index is relatively high.

In the following we will try to make the connection between having a certain degree of decorrelation between the different investment products traded and the advantage to be gained in the training and diversification of portfolios.

METHODOLOGY AND HYPOTHESIS OF THE DYNAMIC MODEL OF INVESTMENT

Presentation of Model

The ability to diversify a standard portfolio is in fact the objective of this part where we build portfolios to assess the potential diversification of these strategies; this study will be made using the dynamic investment model in discrete time combined with the approach for evaluating the empirical probability.

The dynamic model of investment used in this paper assumes that the investor has a utility function with iso-elastic form:

$$u(1+r) = \frac{1}{\gamma}(1+r)^{\gamma} \quad (1)$$

Where r is the return of a period and γ is a parameter of risk aversion. (γ = 1 is a neutral risk).

This iso-elastic function assumes that the coefficient of relative risk aversion (RRA) is independent of wealth and that the coefficient of risk aversion γ must be less than 1 (γ <1) for that the coefficient of absolute risk aversion cannot increase with wealth. A simple calculation shows that the coefficient of relative aversion towards risk RRA is equal to the constant (1 - γ).

This function is a special case when γ tends to 0; in fact, it is a logarithmic function of the form:

$$\ln u\,(1 + r) = \ln (1+r) \tag{2}$$

The use of the power utility function or logarithmic model in a multi periodic involves solving a nonlinear optimization problem with constraint. For this the investor must choose a level of risk aversion through the parameter γ in equation (1). In our study γ takes the following values: -50; -30; -20; -15; -10; -5; -3; -1; -0.5; +0.5; and +1. These values cover respectively investors the most averse (γ = -50) to risk neutral (γ = 1).

At the beginning of each period t, the investor chooses a portfolio Φt based on level of risk aversion and the set of constraints. This comes down to solve the optimization problem in each period t:

$$\max_{\Phi_t} E\left[\frac{1}{\gamma}(1 + r_t(\Phi_t))^{\gamma}\right] = \max_{\Phi_t} \sum \pi_{ts} \frac{1}{\gamma}(1 + r_t(\Phi_t))^{\gamma} \tag{3}$$

$$\Phi_{it} \geq 0, \text{ for all } i \quad \textbf{(4)} \quad \text{et} \quad \Sigma\, \Phi_{it} = 1 \tag{5}$$

with:
- ✓ R_{its} (Φ_t): The ex-ante profitability of period t if state s occurs, with: $r_{ts}(\Phi_t) = \sum_i \Phi_{it} r_{its}$
- ✓ γ: The parameter of risk aversion that remains fixed over time
- ✓ Φ_{it}: The proportion of wealth invested in the risky asset i at time t
- ✓ Φ_t: The weight vector
- ✓ R_{it}: The anticipated total cost of asset i at time t
- ✓ π_{ts}: The probability of state s at the end of period t

The constraint (4) eliminates the possibility of short selling while the constraint (5) represents the budget constraint. The value of the inputs of the model is based on an estimation method explained below.

Using the weights chosen for each asset class at the beginning, the profitability of the portfolio performed in month t is selected and recorded. This cycle is repeated for all months.

The application of the model discussed above poses the problem of estimating the distribution of returns. We apply an estimation procedure used by Grauer and Hakansson (1985, 1986, 1987, 1995 and 2001) called the empirical probability assessment approach (EPAA). With this approach an estimation window of T sub-periods is formed.

An estimation window of 24 periods is composed (T = 24), the vector of realized returns for 24 months is calculated, r_{t-j} (j = 1, ..., 24). Each monthly return from this window of estimation has a weight of $\frac{1}{T}$. Thus, the estimation of the profitability of the period T is:

$$E_{t-1}[r_t] = \frac{1}{24} \sum_{j=1}^{24} r_{t-j}$$

Therefore, with this approach for evaluating the empirical probability (EPAA), the estimated returns are obtained on a mobile base and used in a raw form without any adjustment. Another advantage of this approach is the use of the utility function that requires the specification of the entire return distribution used so that there is no loss of information, all periods and all correlations are taken into account.

The returns are calculated for different asset classes independently of each other.

This management program is applied to the period of January 1994 to December 2010. For each level of risk aversion, and for each portfolio, the optimal compositions are calculated every year. The out-of sample returns of the portfolios are then calculated by applying the weights obtained in the returns actually observed. For each portfolio and each level of risk aversion, we thus obtain a distribution of returns out-of sample. These distributions of returns allow us to compare the performance of different portfolios. In our study, we examine three portfolios with different compositions:

1) Portfolio of traditional assets (not including Hedge Fund assets) noted later P1.
2) Portfolio of traditional assets with hedge fund indices generally noted after P2.
3) Traditional asset portfolio with the ten indices of hedge fund strategies noted later P3.

For each level of risk aversion (γ), we calculated the optimal portfolio composition. Remember that the optimal portfolio is the portfolio that maximizes expected utility on the window of 24-month study as specified in the EPAA. The parameters of the different assets are re-estimated every month; our portfolio is reconstructed at the same frequency. We then applied the optimal composition of the period (January 1994-December 2010). We can analyze the performance of the optimal portfolio for each level of given risk aversion.

We present, in the following paragraphs: the synthesis of the optimization program.

Composition of Optimal Portfolios

The tables in appendix 3 represent the average proportions invested in each asset for our three portfolios depending on the level of risk aversion. They also show the variability in proportions of one month to another of the study period. The proportions shown represent the averages of optimal proportions that satisfy the objective of maximizing the expected utility of the investor on a sliding window of 24 months for the three portfolios. The variance presented in the three tables shows the variability of the proportions invested in each asset.

These tables show that investors are more risk averse, the more the proportions of the capital invested in hedge funds are more important. This proportion is very important for high levels of risk aversion (up to 15% and 65% for P2 P3).

We also notice that for a given level of risk aversion for portfolios P2 and P3, the proportion of capital that is invested in hedge funds is more important in the portfolio P3 than P2; this could be explained by the diversity statistics between different hedge fund strategies.

For further clarification of results we will proceed in two stages: First, we study the returns and the Sharpe ratio of the optimal portfolio obtained, we can compare these ratios to see the efficiency of the portfolio from the perspective of conventional finance. In a second step, we will proceed to study the performance of optimal portfolios using the semi-variance and the ratio E (R) / SVM to see if the results are always the same.

Comparison of Returns and Sharpe Ratio of Optimal Portfolios

Table 1 in appendix n°4 summarizes the average of returns in each optimal portfolio: P1, P2 and P3 for every level of risk aversion and volatility.

The observation of the previous table shows that the returns are decreasing with the increase in risk aversion of the investor. Regarding the volatility of optimal portfolios, we observe that the standard deviation of portfolio that does not include hedge funds is more important than the other wallets for the same level of risk aversion.

For the same level of risk aversion, the lowest variance is observed for the portfolio containing both traditional assets and hedge fund strategies.

Comparing the Sharpe ratios of the three optimal portfolios shows that the optimal portfolio that contains traditional assets and the strategies of hedge funds outperforms the other two portfolios in high level of risk aversion (with some exceptions).

Comparison of the Average / Semi-Variance Optimal Portfolios

Table 2 in appendix n°4 presents a summary of the analysis of the performance of optimal portfolios through the semi-variance.

The comparison of the semi-variance in the portfolios P1 and P2 indicates that P2 realizes for all levels of risk aversion, a lower semi-variance than P1. The diversified portfolio of assets in traditional indices and hedge fund strategies (P3) has the lowest semi-variance for each level of risk aversion, compared to portfolios P1 and P2. The semi-variance criterion does not allow us to conclude on the performance of portfolios or classify them. Indeed, for the same level of risk aversion, the portfolios, which have the lowest semi-variance, also hold the lowest average return. For this reason we present the ratio E(R)/SVM that takes into account these two dimensions namely the average of return and the semi-variance SV_M.

The observation of the ratio E (R) / SV_M in the optimal portfolio containing the traditional assets and indices of hedge fund strategies shows that this ratio is consistently higher in P3 than for portfolios P1 and P2 at all levels of risk aversion. We can conclude after this result that the portfolio P3 is better than P1 and P2. So a diversified portfolio with both traditional assets and hedge fund strategies is more effective than a diversified portfolio only in the traditional asset or in classic assets with the general index of hedge fund.

This finding confirms our hypothesis of the gain of diversification brought by the indices of hedge fund strategies to a portfolio of traditional assets.

CONCLUSION

Using our database we studied, initially, the correlation between the investment strategies used by hedge fund managers with various traditional benchmarks. We conducted an analysis by strategy, sub-strategy and considering the funds individually. This study was not only to analyze the correlation of hedge funds with equity indices, bonds and commodities as a whole, as had been able to do Edwards and Caglayan (2000) for example, but rather to see which indices each strategy could be linked. In addition, we analyzed the entire period for which data focused, then we looked at eight sub periods. The results allow the following conclusions:

- The majority of sub-strategies are correlated with the same ratings as their strategies of belonging.
- There is a significant difference between the strategies correlations with indices, and those of the individual funds with the same indices. The second is much lower; the investor may not be based on aggregate results for the hedge fund industry or in any strategies before making his investment;
- The correlation of hedge funds with conventional indices tends to increase during periods of instability or crisis. The hedge funds then lose some of their advantage of decorrelation. In some cases they may even be highly correlated with classic indices.

The results show the complexity of the hedge fund industry. Hedge funds are not a homogeneous class of assets, which are thought usually to have low correlation with financial markets.

In the second part of the study and to compare the contribution of hedge funds in a diversified portfolio, we analyzed the ex-post performance of managed portfolios that offer maximum expected utility of the 24 months preceding the choice portfolio, here based on historical asset returns. Indeed, the use of an iso-elastic utility function in the dynamic model of investment replaces the traditional portfolio optimization by the mean-variance, since recent studies show that hedge fund returns are not normally distributed. In addition, the EPAA enables us to incorporate all the information and the moments of the distribution of returns.

Our study has shown the gain offered by the addition of hedge fund strategies to a portfolio of stocks and traditional assets, this gain is more significant when we use a set of indices of hedge fund strategies instead a general index of hedge funds. Indeed, the diversity indices of hedge fund strategies offer a better choice for investors.

In this research, some issues have not been extensive, including the problem of Skewness and Kurtosis. Indeed, Kat, Lu and Davies (2006) point out that the use of a utility function represents an alternative approach to optimize a portfolio containing hedge funds and thus circumvent the problem of using the mean-variance optimization. According to these authors, this utility function has the three following properties: (1) non-satiation (2) risk aversion (3) risky assets are inferior goods. By cons, this function does not provide an exact order of preference of portfolio risk using the three first moments of the portfolio's return. Kat, Lu and Davies (2006) then propose to isolate the effect of every moment. An interesting extension of our study would take into account the work of Kat, Lu and Davies (2006) which proposes the use of an optimization model mean-variance-Skewness-Kurtosis.

ENDNOTES

1. **CS:** Credit Suisse/Tremont Hedge Fund Index **CA:** Convertible Arbitrage **DSB:** Dedicated Short Bias **EM:** Emerging Markets **EMN:** Equity Market Neutral **EV:** Event Driven **D EV:** Distressed **MS EV:** Multi-Strategy **RA EV:** Risk Arbitrage **FIA:** Fixed Income Arbitrage **GM:** Global Macro **LES:** Long/Short Equity **MF:** Managed Futures **MS:** Multi-Strategy

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APPENDIX

APPENDIX 1
TABLE 1
DESCRIPTIVE STATISTICS OF HEDGE FUNDS AND CONVENTIONAL INDICES

		Mean average	T-stat	Standard deviation	Median	Min	Max	Sharpe ratio
Part 1	**CS**	0.009	5.002	0.022	0.008	-0.076	0.086	0.268
	CA	0.007	6.741	0.013	0.010	-0.047	0.036	0.354
	DSB	-0.000	-0.095	0.049	-0.004	-0.087	0.227	0.202
	EM	0.008	2.160	0.048	0.014	-0.230	0.164	2.161
	EMN	0.008	11.852	0.008	0.008	-0.012	0.033	1.125
	EV	0.009	7.210	0.016	0.010	-0.118	0.037	1.073
	D EV	0.011	7.407	0.018	0.012	-0.125	0.041	1.188
	MS EV	0.009	6.232	0.017	0.008	-0.115	0.080	1.530
	RA EV	0.006	5.954	0.012	0.006	-0.062	0.039	1.475
	FIA	0.005	5.882	0.011	0.007	-0.070	0.021	0.472
	GM	0.010	4.016	0.032	0.011	-0.116	0.106	0.617
	LES	0.011	4.143	0.033	0.009	-0.114	0.210	0.582
	MF	0.004	1.528	0.036	0.002	-0.147	0.100	1.176
	MS	0.008	7.753	0.012	0.008	-0.048	0.036	0.430
Part 2	**SP**	0.008	2.430	0.041	0.012	-0.146	0.097	0.430
	MSCI	0.007	2.123	0.039	0.011	-0.135	0.089	0.466
	JPM	0.007	0.854	0.104	0.009	-0.445	0.317	0.159
	LH	0.021	1.989	0.135	0.025	-0.566	0.364	0.127
	GD	0.008	1.916	0.050	0.008	-0.146	0.142	0.636

TABLE 2
CORRELATION BETWEEN HEDGE FUNDS STRATEGIES AND CONVENTIONAL INDICES

	SP	MSCI	JPM	LH	GD	Min	Max
CS	**0.477****	**0.488****	**0.272****	-0.057	**0.080**	**-0.057**	**0.488**
CA	**0.153***	**0.133***	**0.170***	-0.065	**0.010**	**-0.065**	**0.170**
DSB	-0.753**	-0.745**	-0.420**	**0.066**	-0.077	**-0.753**	**0.066**
EM	**0.464****	**0.493****	**0.375****	-0.040	**0.081**	**-0.040**	**0.494**
EMN	**0.359****	**0.329****	**0.308****	**0.004**	**0.055**	**0.004**	**0.359**
EV	**0.550****	**0.578****	**0.439****	-0.112	**0.141***	**-0.112**	**0.578**
D EV	**0.543****	**0.556****	**0.406****	-0.088	**0.093**	**-0.088**	**0.556**
MS EV	**0.431****	**0.479****	**0.394****	-0.138*	**0.188****	**-0.136**	**0.479**
RA EV	**0.405****	**0.406****	**0.374****	-0.002	**0.112**	**-0.002**	**0.406**
FIA	**0.033**	-0.013	**0.050**	-0.052	**0.028**	**-0.052**	**0.050**
GM	**0.194****	**0.133***	**0.125**	-0.010	-0.064	**-0.064**	**0.194**
LES	**0.533****	**0.599****	**0.259****	-0.127	**0.175***	**-0.127**	**0.599**
MF	-0.156*	-0.121	-0.098	**0.097**	**0.071**	**-0.156**	**0.097**
MS	**0.099**	**0.179***	-0.015	-0.067	**0.060**	**-0.067**	**0.179**
Min	**-0.753**	**-0.745**	**-0.420**	**-0.136**	**-0.077**		
Max	**0.550**	**0.599**	**0.439**	**0.097**	**0.188**		

TABLE 3
THE CORRELATION COEFFICIENTS OF ALTERNATIVE STRATEGIES

Corrélations (Tau-B de Kendall)

	C	CA	DSB	EM	EMN	EV	D	D EV	MS EV	RA	GM	LES	MF	MS
CS		0.222*	-	0.458**	0.199**	0.515**	0.419**	0.498**	0.310**	0.259*	0.518*	0.602**	0.174*	0.254**
CA			-	0.175**	0.302**	0.374**	0.311**	0.395**	0.280**	0.319*	0.148*	0.190**	-0.049	0.248**
DSB				-	-	-	-	-	-	-	-	-	-0.015	-
EM					0.200**	0.478**	0.394**	0.483**	0.267**	0.197*	0.290*	0.426**	0.025	0.140**
EMN						0.299**	0.266**	0.305**	0.214**	0.227*	0.153*	0.255**	0.054	0.193**
EV							0.741**	0.668**	0.398**	0.317*	0.250*	0.551**	0.063	0.244**
D EV								0.446**	0.311**	0.251*	0.200*	0.449**	0.072	0.149**
MS									0.366**	0.323*	0.276*	0.483**	0.026	0.328**
RA										0.172*	0.178*	0.379**	0.014	0.149**
FIA											0.241*	0.198**	0.041	0.137**
GM												0.226**	0.232*	0.069
LES													0.083	0.214**
MF														0.047
MS														

** Correlation is significant at the 0.01 level (one-tailed).
* Correlation is significant at the 0.05 level (one-tailed).

TABLE 4
THE PERFORMANCE OF DIFFERENT HEDGE FUND INDICES AND STRATEGIES FOR EACH PERIOD ANALYZED

	1/94-12/95	1/96-12/97	1/98-12/99	1/00-12/01	1/02-12/03	1/04-12/05	1/06-12/07	1/08-12/10
CS	0.660%	1.852%	0.917%	0.398%	0.730%	0.696%	1.42%	1.42%
CA	0.300%	1.258%	0.450%	1.463%	0.681%	0.352%	1.13%	2.09%
DSB	0.325%	-0.114%	-0.679%	0.634%	-0.855%	0.298%	1.83%	2.31%
EM	-0.100%	2.328%	-0.145%	0.076%	1.378%	1.224%	1.76%	1.78%
EMN	0.355%	1.229%	1.124%	0.961%	0.586%	0.650%	1.52%	2.28%
EV	0.745%	1.641%	0.679%	0.754%	0.780%	0.959%	1.62%	1.58%
D EV	1.010%	1.757%	0.818%	0.852%	0.927%	1.058%	1.53%	1.96%
MS EV	0.725%	1.650%	0.526%	0.747%	0.886%	0.757%	2.60%	3.10%
RA EV	0.485%	0.939%	0.736%	0.811%	0.218%	0.411%	1.19%	1.40%
FIA	0.270%	0.995%	0.281%	0.580%	0.557%	0.811%	0.67%	0.41%
GM	0.810%	2.018%	0.183%	1.195%	1.272%	0.856%	1.22%	0.81%
LES	1.360%	1.512%	2.399%	-0.003%	0.567%	0.219%	1.18%	0.86%
MF	-0.395%	0.647%	0.634%	0.316%	1.337%	0.721%	0.46%	0.22%
MS	0.365%	1.259%	0.695%	0.671%	0.886%	0.333%	1.15%	0.72%
SP	1.200%	1.983%	1.868%	-0.891%	-0.003%	0.635%	-0.75%	-4.28%
MSCI	0.896%	1.071%	1.857%	-1.335%	0.257%	1.236%	-0.97%	-0.70%
JPM	1.800%	2.801%	0.045%	-2.115%	0.836%	1.070%	4.25%	0.49%
LH	1.690%	5.231%	2.358%	1.225%	1.206%	2.270%	2.60%	1.95%
GD	0.697%	-0.180%	0.424%	-0.496%	1.876%	-0.180%	0.79%	-1.93%

APPENDIX 2
STUDY OF CORRELATIONS

January 1994 December1995

	SP	MSCI	JPM	LH	GD
CS	0.453*	0.364*	0.352*	0.176	-0.050
CA	0.399*	0.125	0.393*	0.076	0.116
DSB	-0.702**	-0.516*	-0.376*	0.088	0.199
EM	0.200	0.098	0.238	0.144	-0.135
EMN	0.165	0.081	0.225	0.140	-0.069
EV	0.587*	0.407*	0.420*	-0.132	0.192
D EV	0.720**	0.575*	0.359*	-0.159	0.038
MS	0.197	0.176	0.255	-0.220	0.353*
RA	0.032	-0.323	0.205	0.469*	-0.378*
FIA	0.259	-0.138	0.491**	0.268	-0.047
GM	0.236	-0.100	0.455*	0.518**	-0.057
LES	0.503**	0.581*	0.185	-0.437*	0.099
MF	-0.359*	-0.263	-0.141	0.417*	-0.143
MS	0.073	0.259	-0.020	-0.334	0.248

January 1996 December1997

	SP	MSCI	JPM	LH	GD
CS	0.758**	0.631*	0.611**	0.066	-0.067
CA	0.094	0.190	0.542**	-0.051	0.105
DSB	-0.646**	-0.650*	-0.467*	0.012	-0.358*
EM	0.529**	0.554*	0.538**	0.103	-0.282
EMN	0.686**	0.619*	0.681**	0.013	0.183
EV	0.711**	0.708*	0.652**	-0.028	0.098
D EV	0.684**	0.668*	0.621**	0.048	0.197
MS	0.629**	0.624*	0.579**	-0.086	0.004
RA	0.212	0.295	0.233	-0.166	-0.107
FIA	0.302	0.383*	0.273	0.015	-0.018
GM	0.625**	0.484*	0.367*	0.137	-0.279
LES	0.690**	0.654*	0.558**	-0.102	0.382*
MF	0.566**	0.552*	0.573**	0.010	-0.065
MS	-0.300	-0.298	-0.231	0.598**	-0.039

January 1998 December1999

	SP	MSCI	JPM	LH	GD
CS	0.542**	0.594**	0.199	-0.363*	0.052
CA	0.191	0.246	0.178	-0.240	0.078
DSB	-0.872**	-0.871**	-0.407*	0.279	-0.218
EM	0.640**	0.741**	0.457*	-0.399*	0.357*
EMN	0.643**	0.681**	0.563**	-0.143	0.204
EV	0.685**	0.762**	0.522**	-0.313	0.270
D EV	0.670**	0.732**	0.436*	-0.245	0.245
MS	0.654**	0.743**	0.573**	-0.366*	0.270
RA	0.656**	0.697**	0.534**	-0.207	0.349
FIA	0.015	-0.029	-0.050	-0.263	-0.104
GM	0.194	0.218	-0.066	-0.288	-0.194
LES	0.813**	0.838**	0.321	-0.291	0.169
MF	-0.317	-0.351*	-0.365*	0.126	-0.135
MS	-0.210	-0.213	-0.327	-0.331	-0.212

January 2000 December 2001

	SP	MSCI	JPM	LH	GD
CS	0.228	0.353*	0.151	0.111	0.430*
CA	0.191	0.170	0.000	0.036	0.252
DSB	-0.775**	-0.795*	-0.301	-0.213	-0.064
EM	0.618**	0.671*	0.359*	0.347*	0.152
EMN	0.510**	0.487*	0.354*	0.098	0.091
EV	0.381*	0.453*	0.190	0.179	0.453*
D EV	0.284	0.289	0.151	0.056	0.075
MS	0.331	0.435*	0.157	0.246	0.590*
RA	0.175	0.255	0.140	0.171	0.563*
FIA	0.036	0.097	-0.072	0.143	0.227
GM	-0.055	0.050	0.268	-0.046	0.179
LES	0.235	0.356*	0.089	0.127	0.429*
MF	-0.453*	-0.410*	0.022	0.066	-0.042
MS	0.328	0.408*	0.118	0.102	0.423*

January 2002 December 2003

	SP	MSCI	JPM	LH	GD
CS	0.534**	0.595**	0.599**	0.144	-0.039
CA	0.170	0.149	0.330	0.140	-0.085
DSB	-0.850**	-0.850**	-0.778**	0.050	0.015
EM	0.781**	0.793**	0.608**	-0.040	-0.131
EMN	-0.019**	0.019**	0.022**	0.042	-0.074
EV	0.590**	0.609**	0.638**	0.082	-0.132
D EV	0.553**	0.567**	0.622**	0.098	-0.140
MS	0.579**	0.621**	0.571**	0.292	-0.163
RA	0.597	0.677	0.634	0.260	-0.035
FIA	-0.228	-0.152	-0.009	-0.191	0.075
GM	-0.075	-0.062	-0.073	-0.043	0.045
LES	0.653**	0.735**	0.671**	0.197	-0.109
MF	-0.418*	-0.394*	-0.328	0.066	0.442*
MS	0.579*	0.621*	0.571*	0.292	-0.163

January 2004 December 2005

	SP	MSCI	JPM	LH	GD
CS	0.567**	0.855**	0.153	-0.201	0.217
CA	0.278	0.453**	0.111	-0.170	0.012
DSB	-0.741**	-0.749**	-0.315*	0.212	0.155
EM	0.446**	0.730**	0.386*	-0.097	0.210
EMN	0.048	0.287*	0.484**	-0.219	0.176
EV	0.586**	0.815**	-0.031	-0.169	0.143
D EV	0.466**	0.753**	-0.012	-0.240	0.226
MS	0.444**	0.750**	0.066	-0.233	0.234
RA	0.627**	0.789**	0.146	-0.230	0.092
FIA	0.029	0.227	-0.106	-0.044	0.311*
GM	0.263	0.559**	0.129	-0.188	0.338*
LES	0.653**	0.908**	0.241	-0.246	0.123
MF	0.535**	0.643**	0.184	0.007	0.175
MS	0.447**	0.731**	0.052	-0.264	0.249

January 2006 December 2007

	SP	MSCI	JPM	LH	GD
CS	-0.130	0.093	0.313	0.173	0.211
CA	0.087	0.097	0.144	-0.097	0.016
DSB	-0.775**	-0.789**	-0.386	0.086	-0.079
EM	0.508**	0.580**	0.431*	-0.127	0.189
EMN	0.149	0.144	0.240	-0.003	0.017
EV	0.450*	0.489*	0.421*	-0.119	0.122
D EV	0.456*	0.480*	0.401*	-0.099	0.086
MS	0.305*	0.342*	0.403*	-0.116	0.136
RA	0.401*	0.422*	0.408*	-0.034	0.132
FIA	-0.041	-0.014	-0.029	-0.140	0.038
GM	0.078	0.120	0.032	-0.171	-0.043
LES	0.533**	0.605**	0.271	-0.079	0.176
MF	-0.219	-0.169	-0.140	0.065	0.108
MS	0.126	0.175	0.021	-0.088	0.056

January 2008 December 2010

	SP	MSCI	JPM	LH	GD
CS	0.144	0.144	-0.144	-0.144	0.144
CA	0.098	0.098	-0.098	-0.098	0.098
DSB	0.345*	0.345*	-0.345*	-0.345*	0.345*
EM	0.281	0.281	-0.281	-0.281	0.281
EMN	0.501**	0.501**	-0.501	-0.501	0.501**
EV	0.203	0.203	-0.203	-0.203	0.203
D EV	0.264	0.264	-0.264	-0.264	0.264
MS	0.075	0.075	-0.075	-0.075	0.075
RA	0.149	0.149	-0.149	-0.149	0.149
FIA	0.007	0.007	-0.007	-0.007	0.007
GM	-0.088	-0.088	0.088	0.088	-0.088
LES	-0.039	-0.039	0.039	0.039	-0.039
MF	0.022	0.022	-0.022	-0.022	0.022
MS	-0.178	-0.178	0.178	0.178	-0.178

APPENDIX 3
CALCULATION OF THE AVERAGE COMPOSITION

Average composition of a portfolio of traditional portfolio without hedge funds

		SP	MSCI	JPM	LH	GD	Return	Utility
1	**Average**	27.024%	19.754%	19.094%	14.977%	19.152%	**0.904%**	**0.926**
1	**Variance**	0.304%	0.442%	0.343%	0.115%	0.150%	**0.009%**	**0.076**
0.5	**Average**	27.001%	19.782%	19.089%	14.955%	19.174%	**0.934%**	**1.843**
0.5	**Variance**	0.304%	0.444%	0.347%	0.116%	0.152%	**0.009%**	**0.305**
-0.5	**Average**	27.020%	19.776%	19.083%	14.948%	19.174%	**0.933%**	**-1.824**
-0.5	**Variance**	0.306%	0.445%	0.348%	0.116%	0.152%	**0.009%**	**0.306**
-1	**Average**	27.030%	19.773%	19.080%	14.945%	19.172%	**0.933%**	**-0.908**
-1	**Variance**	0.306%	0.445%	0.348%	0.116%	0.153%	**0.009%**	**0.077**
-3	**Average**	27.065%	19.762%	19.067%	14.936%	19.171%	**0.933%**	**-0.297**
-3	**Variance**	0.308%	0.447%	0.350%	0.116%	0.153%	**0.009%**	**0.009**
-5	**Average**	27.098%	19.751%	19.057%	14.926%	19.168%	**0.932%**	**-0.175**
-5	**Variance**	0.310%	0.448%	0.352%	0.116%	0.154%	**0.009%**	**0.003**
-10	**Average**	27.172%	19.727%	19.033%	14.907%	19.160%	**0.932%**	**-0.083**
-10	**Variance**	0.315%	0.452%	0.356%	0.116%	0.154%	**0.009%**	**0.001**
-15	**Average**	27.237%	19.705%	19.015%	14.889%	19.155%	**0.932%**	**-0.053**
-15	**Variance**	0.318%	0.455%	0.359%	0.116%	0.155%	**0.009%**	**0.000**
-20	**Average**	27.292%	19.687%	18.999%	14.874%	19.149%	**0.931%**	**-0.038**
-20	**Variance**	0.322%	0.457%	0.361%	0.116%	0.156%	**0.009%**	**0.000**
-30	**Average**	27.385%	19.654%	18.975%	14.850%	19.137%	**0.931%**	**-0.023**
-30	**Variance**	0.327%	0.461%	0.365%	0.116%	0.157%	**0.009%**	**0.000**
-50	**Average**	27.518%	19.605%	18.943%	14.816%	19.118%	**0.931%**	**-0.012**
-50	**Variance**	0.335%	0.467%	0.370%	0.116%	0.158%	**0.009%**	**0.000**

R: rentability of portfolio

Average composition of an optimal portfolio of traditional assets with a general index of hedge funds

		CS	SP	MSCI	JPM	LH	GD	Return	Utility
1	**Average**	14.838%	23.242%	17.683%	15.626%	12.537%	15.816%	**0.925%**	**1.009**
1	**Variance**	0.104%	0.000%	0.000%	0.000%	0.000%	0.000%	**0.009%**	**0.000**
0.5	**Average**	14.797%	23.238%	17.671%	15.625%	12.508%	15.833%	**0.924%**	**2.009**
0.5	**Variance**	0.111%	0.000%	0.000%	0.000%	0.000%	0.000%	**0.009%**	**0.000**
-0.5	**Average**	14.804%	23.266%	17.665%	15.616%	12.501%	15.831%	**0.924%**	**-1.991**
-0.5	**Variance**	0.112%	0.000%	0.000%	0.000%	0.000%	0.000%	**0.009%**	**0.000**
-1	**Average**	14.812%	23.282%	17.663%	15.612%	12.496%	15.828%	**0.924%**	**-0.991**
-1	**Variance**	0.112%	0.000%	0.000%	0.000%	0.000%	0.000%	**0.009%**	**0.000**
-3	**Average**	14.822%	23.334%	17.653%	15.595%	12.484%	15.824%	**0.924%**	**-0.32**
-3	**Variance**	0.112%	0.000%	0.000%	0.000%	0.000%	0.000%	**0.009%**	**0.000**
-5	**Average**	14.920%	23.097%	16.638%	14.083%	13.658%	17.603%	**0.884%**	**-0.192**
-5	**Variance**	0.106%	0.000%	0.000%	0.000%	0.000%	0.000%	**0.009%**	**0.000**
-10	**Average**	15.111%	22.498%	15.787%	13.952%	14.016%	18.925%	**0.913%**	**-0.092**
-10	**Variance**	0.087%	0.000%	0.000%	0.000%	0.000%	0.000%	**0.008%**	**0.000**
-15	**Average**	15.120%	22.541%	15.767%	13.921%	14.001%	18.959%	**0.913%**	**-0.059**
-15	**Variance**	0.087%	0.000%	0.000%	0.000%	0.000%	0.000%	**0.008%**	**0.000**
-20	**Average**	15.122%	22.572%	15.752%	13.893%	13.989%	18.990%	**0.913%**	**-0.042**
-20	**Variance**	0.086%	0.000%	0.000%	0.000%	0.000%	0.000%	**0.008%**	**0.000**
-30	**Average**	15.125%	24.491%	15.598%	14.472%	13.543%	17.099%	**0.947%**	**-0.026**
-30	**Variance**	0.086%	0.000%	0.000%	0.000%	0.000%	0.000%	**0.008%**	**0.000**
-50	**Average**	15.095%	24.300%	17.340%	15.469%	12.370%	15.682%	**0.917%**	**-0.014**
-50	**Variance**	0.083%	0.000%	0.000%	0.000%	0.000%	0.000%	**0.009%**	**0.000**

Average composition of an optimal portfolio of assets with traditional indices of hedge fund strategies (in %)

		CA	DSB	EM	EMN	EV	D EV	MS EV	RA EV	FIA	GM	LES	MF	MS	%
1	**Averag**	5.05	6.21	6.15	4.42	3.61	4.27	3.73	4.58	5.18	4.19	4.87	5.80	4.05	**62.09**
1	**Varianc**	0.01	0.02	0.02	0.01	0.01	0.01	0.01	0.02	0.01	0.02	0.04	0.02	0.00	
0.5	**Averag**	5.12	6.18	6.26	4.48	3.57	4.29	3.74	4.63	5.71	4.28	4.93	5.87	4.11	**63.17**
0.5	**Varianc**	0.01	0.02	0.02	0.01	0.01	0.01	0.01	0.03	0.02	0.02	0.05	0.02	0.01	
-0.5	**Averag**	5.12	6.09	6.26	4.47	3.53	4.26	3.73	4.66	5.41	4.29	4.93	5.83	4.12	**62.72**
-0.5	**Varianc**	0.01	0.01	0.02	0.01	0.01	0.01	0.01	0.03	0.01	0.02	0.06	0.02	0.01	
-1	**Averag**	5.12	6.01	6.26	4.47	3.52	4.25	3.73	4.68	5.44	4.30	4.92	5.81	4.14	**62.65**
-1	**Varianc**	0.01	0.01	0.02	0.01	0.01	0.01	0.01	0.03	0.01	0.02	0.06	0.02	0.01	
-3	**Averag**	5.14	5.91	6.24	4.50	3.55	4.26	3.77	4.73	5.50	4.33	4.93	5.76	4.19	**62.79**
-3	**Varianc**	0.01	0.01	0.02	0.01	0.01	0.01	0.01	0.03	0.01	0.02	0.06	0.02	0.01	
-5	**Averag**	5.16	5.74	6.18	4.53	3.60	4.28	3.83	4.79	5.54	4.35	4.95	5.69	4.26	**62.89**
-5	**Varianc**	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.03	0.01	0.02	0.06	0.02	0.01	
-10	**Averag**	5.19	5.64	6.16	4.57	3.65	4.31	3.89	4.83	5.58	4.39	4.98	5.67	4.32	**63.18**
-10	**Varianc**	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.03	0.01	0.02	0.06	0.01	0.01	
-15	**Averag**	5.22	5.49	6.08	4.61	3.71	4.34	3.93	4.87	5.60	4.42	5.00	5.62	4.36	**63.23**
-15	**Varianc**	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.03	0.01	0.02	0.05	0.01	0.01	
-20	**Averag**	5.33	5.54	6.17	4.72	3.82	4.46	4.05	4.96	5.70	4.53	5.08	5.70	4.47	**64.52**
-20	**Varianc**	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.03	0.01	0.02	0.05	0.01	0.01	
-30	**Averag**	5.35	5.41	6.12	4.71	4.05	4.25	4.37	5.31	5.31	4.85	5.60	5.45	4.64	**65.42**
-30	**Varianc**	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.04	0.01	0.00	
-50	**Averag**	5.34	5.35	6.09	4.72	3.90	4.49	4.09	5.31	5.67	4.73	5.32	5.68	4.52	**65.20**
-50	**Varianc**	0.01	0.01	0.02	0.01	0.01	0.01	0.00	0.02	0.01	0.02	0.04	0.02	0.01	

Next: Average composition of an optimal portfolio of assets with traditional indices of hedge fund strategies

		SP	MSCI	JPM	LH	GD	R	Utility
1	**Average**	6.133%	6.605%	7.602%	8.846%	8.721%	**0.884%**	**1.009**
1	**Variance**	0.033%	0.084%	0.104%	0.177%	0.302%	**0.002%**	**0.000**
0.5	**Average**	6.450%	6.040%	7.707%	9.031%	7.604%	**0.984%**	**2.010**
0.5	**Variance**	0.047%	0.061%	0.120%	0.218%	0.264%	**0.003%**	**0.000**
-0.5	**Average**	6.554%	6.138%	7.711%	9.094%	7.788%	**0.962%**	**-1.990**
-0.5	**Variance**	0.057%	0.072%	0.125%	0.231%	0.254%	**0.003%**	**0.000**
-1	**Average**	6.590%	6.174%	7.704%	9.138%	7.740%	**0.942%**	**-0.991**
-1	**Variance**	0.061%	0.077%	0.134%	0.242%	0.259%	**0.004%**	**0.000**
-3	**Average**	6.632%	6.226%	7.634%	9.096%	7.622%	**0.939%**	**-0.324**
-3	**Variance**	0.064%	0.081%	0.142%	0.245%	0.261%	**0.004%**	**0.000**
-5	**Average**	6.649%	6.264%	7.593%	9.045%	7.564%	**0.938%**	**-0.191**
-5	**Variance**	0.067%	0.085%	0.142%	0.240%	0.261%	**0.004%**	**0.000**
-10	**Average**	6.665%	6.292%	7.475%	8.922%	7.465%	**0.936%**	**-0.091**
-10	**Variance**	0.069%	0.087%	0.136%	0.229%	0.261%	**0.004%**	**0.000**
-15	**Average**	6.632%	6.276%	7.498%	8.913%	7.449%	**0.933%**	**-0.058**
-15	**Variance**	0.067%	0.084%	0.138%	0.226%	0.262%	**0.004%**	**0.000**
-20	**Average**	6.729%	6.382%	6.328%	8.652%	7.395%	**0.929%**	**-0.042**
-20	**Variance**	0.080%	0.100%	0.080%	0.235%	0.259%	**0.004%**	**0.000**
-30	**Average**	5.917%	5.574%	6.576%	8.846%	7.668%	**0.926%**	**-0.026**
-30	**Variance**	0.024%	0.020%	0.065%	0.188%	0.241%	**0.004%**	**0.000**
-50	**Average**	6.023%	5.595%	6.986%	8.242%	7.951%	**0.904%**	**-0.013**
-50	**Variance**	0.014%	0.017%	0.084%	0.137%	0.220%	**0.003%**	**0.000**

APPENDIX 4
TABLE 1
PROFITABILITY AND AVERAGE SHARPE RATIOS OF OPTIMAL PORTFOLIOS

Stock portfolio without hedge funds (P1)
Equity portfolio and general index of Hedge Fund (P2)
Portfolio of stocks and indices of hedge fund strategies (P3)

		1	0.5	-0.5	-1	-3	-5	-10	-15	-20	-30	-50
P1	Average (%)	0.931	0.931	0.931	0.931	0.932	0.932	0.932	0.933	0.933	0.933	0.934
	Standard deviation (%)	0.929	0.929	0.929	0.929	0.929	0.930	0.930	0.930	0.930	0.930	0.931
	Ratio of Sharpe	0.772	0.771	0.771	0.771	0.771	0.771	0.771	0.771	0.770	0.770	0.770
P2	Average (%)	0.917	0.947	0.913	0.913	0.913	0.884	0.924	0.924	0.924	0.924	0.925
	Standard deviation (%)	0.929	0.887	0.888	0.888	0.888	0.927	0.929	0.929	0.929	0.929	0.929
	Ratio of Sharpe	0.763	0.763	0.763	0.763	0.763	0.721	0.786	0.786	0.785	0.824	0.756
P3	Average (%)	0.904	0.926	0.929	0.933	0.936	0.938	0.939	0.942	0.962	0.984	0.884
	Standard deviation (%)	0.543	0.595	0.606	0.614	0.620	0.623	0.620	0.621	0.571	0.530	0.470
	Ratio of Sharpe	1.422	1.451	1.307	1.170	1.166	1.160	1.162	1.169	1.177	1.195	1.268

TABLE 2
COMPARISON OF THE AVERAGE / SEMI-VARIANCE OPTIMAL PORTFOLIOS

		1	0.5	-0.5	-1	-3	-5	-10	-15	-20	-30	-50
P1	Average (%)	0.931	0.931	0.931	0.931	0.932	0.932	0.932	0.933	0.933	0.933	0.934
	Semi variance (% 10^{-2})	59.65	59.57	59.57	59.57	59.56	59.56	59.55	59.55	59.56	59.56	59.58
	E(R) / SV	156.04	156.29	156.34	156.38	156.43	156.50	156.55	156.59	156.59	156.61	156.71
P2	Average (%)	0.917	0.947	0.913	0.913	0.913	0.884	0.924	0.924	0.924	0.924	0.925
	Semi variance (% 10^{-2})	55.68	55.70	55.70	55.70	55.70	53.29	48.09	48.08	48.08	49.98	55.62
	E(R) / SV	164.76	169.93	163.86	163.89	163.96	165.98	192.10	192.18	192.21	184.94	166.21
P3	Average (%)	0.904	0.926	0.929	0.933	0.936	0.938	0.939	0.942	0.962	0.984	0.884
	Semi variance (% 10^{-2})	11.74	13.00	16.63	21.75	21.72	21.80	21.44	20.95	20.37	19.42	17.00
	E(R) / SV	770.03	712.33	558.28	429.05	430.96	430.44	437.80	449.63	472.39	506.82	520.02