Analysis of the Relation Between the Spiders' Spot and Option Implied Volatility

Stoyu I. Ivanov San Jose State University

In this study we use spectral analysis on SPY (Spiders) options to examine the relation between option spot and implied volatility for this exchange trade fund. We attempt to address the question is there a relation between the option spot and implied volatility or option implied volatility has no relation with the spot exchange trade fund volatility. We find that this relation does exist for SPY at-the-money call and put options and the in-the-money call and out-of-the-money put options. Using two spectral statistics – the coherence and the phase statistics, we find that the SPY option implied volatility and spot volatility have a relation and that the SPY option implied volatility.

INTRODUCTION

In this study we address the issue - is there a relation between the spot and implied volatility or option implied volatility has no relation with the spot Exchange Traded Fund (ETF) volatility. The ETF that we choose to study is the Standard & Poor's Depositary Receipts SPDR S&P 500 ETF Trust, with ticker symbol – SPY, and popular name – Spider. The SPY tracks the S&P 500 index and is a Unit Investment Trust. The null hypothesis of the study is SPY spot volatility is not related to the SPY option implied volatility. We extend the work of Canina and Figlewski (1993) and Christensen and Prabhala (1998) who examine the same issue. Both studies use parametric tests to establish if such relation exists. However, financial data is typically non-normally distributed which invalidates conclusions based on parametric tests. In contrast to their studies we use the non-parametric tools of spectral analysis on SPY options to examine the relation between ETF spot and implied option volatility for this ETF.

To the best of our knowledge this is the first study to address this question with non-parametric tools. This helps extend our understanding of ETF option behavior. We find that this relation does exist for the ETF at-the-money call and put options and the ETF in-the-money call and out-of-the-money put categories. Only for these option categories do the coherence spectral statistic is consistently above 50%. Using another spectral statistic, the phase statistic, we also find that the SPY option implied volatility and spot volatility have a lead-lag relation - the SPY option implied volatility leads the SPY spot volatility.

LITERATURE REVIEW

In this study I extend ideas developed and examined by Canina and Figlewski (1993), Christensen and Prabhala (1998), Bakshi, Kapadia and Madan (2003) and Ivanov, Whitworth and Zhang (2011). Both Canina and Figlewski (1993) and Christensen and Prabhala (1998) examine the issue of option implied volatility being related to spot and future spot volatility. Canina and Figlewski (1993) find that S&P 100

(OEX) index options implied volatility does not incorporate spot S&P 100 index volatility information. Christensen and Prabhala (1998) find the opposite. However, both studies use Ordinary Least Squares methodology which is a parametric tool. Financial and option data are typically non-normally distributed which leads to misspecification in parametric models. Therefore, we first show that the data are non-normally distributed and use non-parametric spectral analysis methodology. Additionally, both studies use the Black – Scholes formula to estimate the OEX option implied volatility. However, the OEX option is an American style option, whereas the Black – Scholes formula is used to estimate the implied volatility for European style options. We address and correct this issue as well because the SPY ETF option used in this study is also American style, but we do not use the Black – Scholes formula to estimate its implied volatility. We follow Ivanov, Whitworth and Zhang (2011) methodology and recompute the SPY option implied volatility based on a 100-step binomial tree model.

Bakshi, Kapadia and Madan (2003) study skewness in stock and option markets. They find that individual stocks are more volatile than the index that they belong to and that the reason for the option implied volatility changes might be due to the difference in the stochastic process governing the returns of the underlying indexes and stocks.

Ivanov, Whitworth and Zhang (2011) study ETF option implied volatility and find that volatility smiles of ETF options are more pronounced than for index options. They also find that the reason for the difference is not due to the proposed by Bakshi, Kapadia and Madan (2003) difference in the stochastic processes of the underlying indexes. Their findings are in agreement with a study by Bollen and Whaley (2004).

Bollen and Whaley (2004) study S&P 500 index options and stock options of the underlying stocks on intradaily basis. Bollen and Whaley (2004) find that S&P 500 index option implied volatility are most often influenced by demand for index puts and by demand for call stock options. Bollen and Whaley (2004) interpretation of the behavior of option implied volatilities based on the differential demand for options is different from the interpretation of Bakshi, Kapadia and Madan (2003) that the reason for the implied volatility changes might be due to the difference in the stochastic process governing the returns of the underlying indexes and stocks.

Therefore, based on these studies and the lack of agreement and consistent evidence in the literature we propose the use of non-parametric tools such as spectral analysis to examine if spot volatility is related to option implied volatility.

DATA AND METHODOLOGY

The options data are obtained from deltaneutral.com but modified to account for the fact that ETF options are American options. The data for the SPY are over the period - 01/10/2005 to 12/30/2005. The SPY was introduced on January 30, 1993 and is designed to be 1/10 of the S&P500. The SPY options are listed on the Chicago Board Options Exchange (CBOE) and started trading in January 2005 that is why we focus on the one ear period - 01/10/2005 to 12/30/2005. SPY options are American style options. Considering that SPY options are American style, whereas the implied volatility in the original database are computed based on the Black-Scholes formula which is for European style options, we follow Ivanov, Whitworth and Zhang (2011) methodology and re-compute the SPY option implied volatility based on a 100-step binomial tree model. The null hypothesis of the study is:

H0: SPY spot volatility is not related to the SPY option implied volatility.

Rejection of the null hypothesis would indicate that such a relation exists. Following the studies of Day and Lewis (1988), Xu and Taylor (1994) and Ivanov, Whitworth and Zhang (2011) we filter the options data to minimize influence of outliers in the analysis that follows. The filtering is conducted based on the following standard criteria:

The time to expiration is filtered to be greater than 7 days and less than 30 days.

- a) The option is filtered to satisfy the European option boundary conditions, $c < Se^{-\delta T} Xe^{-rT}$ and $p < Xe^{-rT} - Se^{-\delta T}$.
- b) The option is filtered to satisfy the American option boundary conditions, C < S X and P < X S.
- c) The option is filtered not to be so deep-out of or in-the-money that exercise is either impossible or absolutely certain; i.e., we filter based on the absolute value of the option's hedging delta to be within the bounds 0.02 and 0.98.

To identify moneyness categories we follow the Bollen and Whaley (2004) classification based on the option's delta. The five categories that we examine are identified in Table 1.

 TABLE 1

 BOLLEN AND WHALEY (2004) CLASSIFICATION OF MONEYNESS CATEGORIES

Category	Labels	Range
1	Deep-in-the-money (DITM) call	0.875<∆c<=0.98
	Deep-out-of-the-money (DOTM) put	-0.125<∆p<= -0.02
2	In-the-money (ITM) call	0.625<∆c<=0.875
	Out-of-the-money (OTM) put	-0.375<∆p<= -0.125
3	At-the-money (ATM) call	0.375<∆c<=0.625
	At-the-money (ATM) put	-0.625<∆p<= -0.375
4	Out-of-the-money (OTM) call	0.125<∆c<=0.375
	In-the-money (ITM) put	-0.875<∆p<= -0.625
5	Deep-out-of-the-money (DOTM) call	0.02<∆c<=0.125
	Deep-in-the-money (DITM) put	-0.98<∆p<= -0.875

After we identify the different categories of call and put options we analyze the data by using spectral analysis. We examine if the spot SPY volatility is related to the SPY options implied volatility. Spectral analysis is considered to be a more robust analytic method than traditional regression analysis because it is non-parametric. Spectral analysis does not require a model specification and does not impose structure on the link between dependent and independent variables (Jenkins, 1965). Nevertheless, the nonparametric nature of spectral analysis has inefficiencies because large number of parameters need to be estimated. Additionally, the condition for stationarity of the studied time series is essential to reach any meaningful conclusions.

Consider the covariance stationary random variable 'y_t':

$$y_t = \sum_{j=0}^{\infty} b_j \varepsilon_{t-j} = B(L)\varepsilon_t.$$
⁽¹⁾

B(L) is the polynomial of the lag operator for 'b_i' with:

$$B(L) = b_0 + b_1 L + b_2 L^2 + \dots = \sum_{j=0}^{\infty} b_j L^j,$$
(2)

where 'y_t' is generated from the random white noise process ' ε_t '. ' ε_t ' has the conventional statistical properties:

$$E(\varepsilon_t) = 0, \forall t \tag{3}$$

$$E(\varepsilon_t^2) = \sigma_\varepsilon^2, \forall t \tag{4}$$

$$E(\varepsilon_t \varepsilon_{t-s}) = 0, \forall t \cup \forall s \neq 0$$

the covariance function for y_t leads to:

$$E(y_t y_{t-k}) = \sigma_{\varepsilon}^2 \sum_{j=-\infty}^{\infty} b_j b_{j-k} .$$
⁽⁶⁾

The covariance has a covariance generating function characterized by the following equation:

$$g_{y}(z) = \sigma_{\varepsilon}^{2} B(z) B(z^{-1}) = \sum_{k=-\infty}^{\infty} c_{y}(k) z^{k} , \qquad (7)$$

Further if 'z' is characterized by

$$z = e^{-i\omega}, \tag{8}$$

the covariance generating function is characterized by:

$$g_{y}(e^{-i\omega}) = \sum_{k=-\infty}^{\infty} c_{y}(k)e^{-i\omega k}, (-\pi < \omega < \pi),$$
⁽⁹⁾

this is called the spectrum of the variable ' y_t ' with 'w' being the frequency. The spectrum is the Fourier transform of the covariogram of the examined variable. The spectrum can be further modified to have more useful geometric properties:

$$g_{y}(e^{-i\omega}) = \sum_{k=-\infty}^{\infty} c_{y}(k)e^{-i\omega k} = c_{y}(0) + 2\sum_{k=1}^{\infty} c_{y}(k)\cos(\omega k),$$
(10)

since $\cos(-\omega k) = \cos(\omega k)$ and $e^{-i\omega} = e^{i\omega}$. The equation in (10) represents the link between the function of the covariogram of variable 'y_t' and the cos-function of the frequency and means that the spectrum is always nonnegative. This also means that the spectrum of a white noise process is a constant number.

This univariate analysis can be extrapolated to a bivariate framework, known as co-spectral analysis and it helps to analyze the covariation between two stationary processes ' x_t ' and ' y_t '. The covariance generating function is represented by the equation:

$$g_{yx}(z) = \sum_{k=-\infty}^{\infty} c_{yx}(k) z^k , \qquad (11)$$

and again substituting in $z = e^{-i\omega}$ we get the bivariate spectrum represented by the following equation:

$$g_{yx}(e^{-i\omega}) = \sum_{k=-\infty}^{\infty} c_{yx}(k)e^{-i\omega k} .$$
⁽¹²⁾

(for a detailed discussion of this methodology see Sargent, 1979).

Therefore, in the analysis that follows covariance stationarity tests will be performed first, followed by univariate spectral analysis to establish "typical" spectral shape for the variables, cross spectrum calculations will be used last to establish comovement between the spot SPY volatility and the call and put options implied volatilities.

Following Sargent (1979) and Erol, and Koray (1988) kernels are used to smooth the spectral density of the variables by calculating a weighted moving average of nearby periodogram points to get rid of any noise. The theory does not specify a preference of one kernel over another therefore we use three different kernel specifications in the analysis to establish robustness of the results. Andrews (1991) discusses detailed description of the smoothed periodogram using kernels which is defined as:

$$\hat{J}_{i}(l(q)) = \sum_{\tau = -l(q)}^{l(q)} w(x)\tilde{J}_{i+\tau} .$$
⁽¹³⁾

Where w(x) is the kernel, $x = \frac{\tau}{l(q)}$, l(q) is the bandwidth parameter over which the smoothing will be

performed. The parameter 'q' is the number of periodogram ordinates +1. At the endpoints of the bandwidth a cycle is used to compute averages which is represented by:

$$\widetilde{J}_{i+\tau} = \begin{cases}
J_{i+\tau} & 0 \le i+\tau \le q \\
J_{-(i+\tau)} & i+\tau < 0 \\
J_{q-(i+\tau)} & i+\tau > q
\end{cases}$$
(14)

The Bartlett kernel is specified as follows:

$$Bartlett = w(x) = \begin{cases} 1 - |x| & |x| \le 1\\ 0 & \text{otherwise} \end{cases} \quad \text{with bandwidth } l(q) = \frac{1}{2}q^{\frac{1}{3}}, \tag{15}$$

the Parzen kernel is specified as follows:

$$Parzen = w(x) = \begin{cases} 1 - 6x^2 + 6|x|^3 & 0 \le |x| \le \frac{1}{2} \\ 2(1 - |x|)^3 & \frac{1}{2} \le |x| \le 1 \\ 0 & \text{otherwise} \end{cases}$$
(16) (16)

and the Tukey - Hanning kernel is specified as follows:

$$Tukey - Hanning = w(x) = \begin{cases} (1 + \cos(\pi x))/2 & |x| \le 1\\ 0 & \text{otherwise} \end{cases} \text{ with bandwidth } l(q) = \frac{2}{3}q^{\frac{1}{5}}.$$
 (17)

Next, we use two spectral analysis statistics to estimate the relationship of the options implied volatility and the spot standard deviation - the coherence statistic and the phase statistic. The coherence from spectral analysis is similar to the correlation coefficient. The coherence represents a percentage of explanation of one of the variables variation by the second variables variation, but at different frequencies of the spectrum (the frequencies range from 0 to $3.14159(\pi)$). The spectrum is usually used to establish the most typical frequency in the frequency domain where the coherence has most meaning. The coherence statistic is computed as follows:

$$coherence(\omega) = \frac{\left|g_{yx}(e^{-i\omega})\right|^2}{g_x(e^{-i\omega})g_y(e^{-i\omega})},$$
(18)

and represents the product of two forward and backward looking gain statistics (Jenkins, 1965) and is in the range 0 to 1. The gain statistic is defined as:

$$g_{yx}(e^{-i\omega}) = \cos(\omega) + i \operatorname{qu}(\omega) = \sqrt{\operatorname{co}^{2}(\omega) + \operatorname{qu}^{2}(\omega)} e^{i \tan^{-1} \left[\frac{\operatorname{qu}(\omega)}{\operatorname{co}(\omega)}\right]} = \operatorname{r}(\omega) e^{i\theta(\omega)} \quad . \tag{19}$$

The different elements in the equation are as follows: co(w) is the cospectrum, qu(w) is the quadrature spectrum and 'i' is the imaginary component in the spectral analysis. The phase statistic is considered to measure the shift between the two waves and can be interpreted as a lead-lag relationship of the two series over the frequencies range and may be used to establish statistical causality. The phase statistic is computed as follows:

$$\theta(\omega) = \tan^{-1} \left[\frac{\operatorname{qu}(\omega)}{\operatorname{co}(\omega)} \right].$$
(20)

However, keep in mind that Hause (1971) and Stone (1975) suggest in the spectral analysis literature that the phase statistic cannot be used to establish lead-lag relationships.

ANALYSIS

Table 2 provides summary statistics on the five different categories of call and put options implied volatilities and the twenty day moving average rolling standard deviations. Over the examined period there are 204 observations. The twenty day mean standard deviation is 1.07. The Deep-in-the-money (DITM) call and put options mean implied volatilities are highest among the five moneyness categories.

TABLE 2

	Category 1		Categ	gory 2	Category 3		Category 4		Category 5		20 day
	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	
	IVc	IVp	IVc	IVp	IVc	IVp	IVc	IVp	IVc	IVp	stdev
Mean	0.26	0.19	0.14	0.14	0.11	0.12	0.11	0.16	0.12	0.38	1.07
Median	0.25	0.19	0.14	0.14	0.11	0.12	0.10	0.13	0.12	0.36	1.00
Std Dev	0.10	0.03	0.02	0.02	0.02	0.02	0.01	0.07	0.02	0.14	0.36
Minimum	0.11	0.14	0.09	0.10	0.08	0.09	0.08	0.08	0.09	0.08	0.40
Maximum	0.76	0.26	0.24	0.18	0.17	0.17	0.15	0.43	0.19	0.84	1.96
Skewness	1.23	0.39	1.02	0.34	1.00	0.19	0.86	1.63	0.58	0.66	0.52
Kurtosis	3.46	-0.38	1.39	-0.18	0.94	-0.44	0.58	2.29	-0.23	1.13	-0.43
20 day											
stdev											
Correlation	0.17	0.27	0.31	0.23	0.31	0.12	0.29	-0.10	0.14	-0.08	1

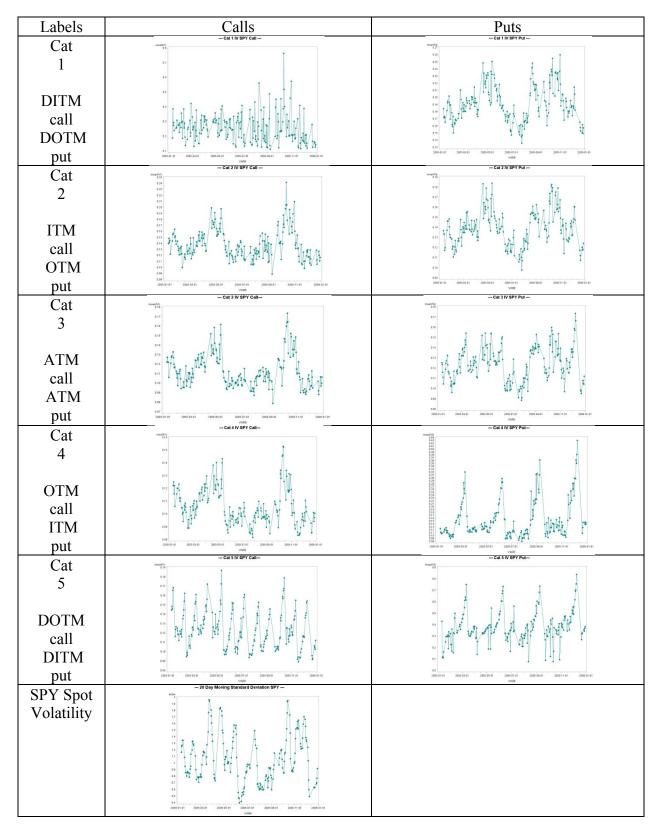
SUMMARY STATISTICS, IMPLIED VOLATILITY OF SPY CALL AND PUT OPTIONS FOR THE PERIOD JANUARY 10, 2005 TO DECEMBER 30, 2005

The mean DITM call option implied volatility is 0.26, whereas the DITM put option mean implied volatility is 0.38. All of the rest of the mean implied volatilities are less than 0.19 and much less than the mean twenty day standard deviations. This suggests that the factors forming option implied volatilities might be different than the spot volatility, motivating and providing evidence that this study is needed, and evidence against the null hypothesis of the study of spot volatility influencing option implied volatility.

In the table correlation coefficients are also provided with the twenty day standard deviations. All of the correlation coefficients are equal or less than 0.31, which also provides evidence against the null hypothesis of the study of spot volatility influencing option implied volatility. However, keep in mind that correlation coefficients might lead to erroneous conclusions because the data are non-normally distributed.

Spectral analysis requires stationarity in the time series, without stationarity in the time series spectral analysis the results of the analysis are meaningless. To examine in more detail the implied volatility and standard deviation distributions, we also compute the third and fourth moments for the variables. These variables are also reported in the table. The results for the skewness indicate that the distributions of both implied volatility and spot standard deviation are positively skewed, which suggests difference from normal distribution. The results for kurtosis, similar to the skewness results, suggest non-normal distribution. The non-normality indicates that parametric tests results would be meaningless. However, normality is not essential for conducting spectral analysis, stationarity is, considering that spectral analysis is a non-parametric methodology. Therefore, stationarity of the call and put options implied volatilities needs to be established first.

FIGURE 1 SPY SPOT AND OPTION IMPLIED VOLATILITY



Visually all option implied volatility series and twenty day moving standard deviation series are stationary as shown visually in Figure 1. However, stationarity needs to be tested formally. The tests for stationarity that we use in the analysis are the Augmented Dickey Fuller (ADF) test for unit roots and White Noise Tests - Fisher's Kappa and Bartlett's Kolmogorov-Smirnov. Fisher's Kappa tests if the largest value of the periodogram J_k is statistically different from the mean value of the periodogram. The basic idea is that if J_k is a white noise process it would have a constant mean and constant variance at any period of the periodogram. The Kolmogorov-Smirnov test examines if the normalized cumulative periodogram of J_k represented by:

$$F_{j} = \frac{\sum_{k=1}^{j} J_{k}}{\sum_{k=1}^{m} J_{k}}; j = 1, 2, 3, ..., m - 1; \begin{cases} m = \frac{n}{2}, \text{ if n is even} \\ m = \frac{n-1}{2}, \text{ if n is odd} \end{cases}$$

is statistically different from the cumulative distribution function of a uniform (0,1) random variable. The null hypothesis of the Kolmogorov-Smirnov test is: the periodogram is a white noise process.

The stationarity test results are presented in Table 3. Based on the stationarity tests results that the call and put options implied volatilities and standard deviations are stationary we can proceed with the spectral analysis.

TABLE 3

WHITE NOISE TEST RESULTS USING AUGMENTED DICKEY FULLER (ADF) TEST FOR UNIT ROOTS (P-VALUES REPORTED), FISHER'S KAPPA (WITH AN APPROXIMATE CRITICAL VALUE OF AROUND 9.707) AND BARTLETT'S KOLMOGOROV-SMIRNOV (BKS) STATISTIC (P-VALUES REPORTED) ON IMPLIED VOLATILITIES AND 20 DAY SPOT STANDARD DEVIATION

	Cat 1		Cat 2		Cat 3		CaT 4		Cat 5		Stdev
	call	put									
ADF Zero Mean	0.0291	0.5751	0.4703	0.5739	0.5267	0.5197	0.5380	0.1794	0.3822	0.1564	0.1722
ADF Single Mean	0.0001	0.0020	0.0014	0.0014	0.0015	0.0014	0.0017	0.0014	0.0014	0.0014	0.0014
ADF Trend	0.0001	0.0155	0.0029	0.0048	0.0106	0.0006	0.0102	0.0041	0.0006	0.0006	0.0006
Kappa	5.056	44.534	32.169	44.292	32.432	22.919	25.262	45.195	41.003	33.966	23.980
BKS	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001

Next we determine the SPY call and put option implied volatility spectral density. For the different moneyness categories the periodicity cycles are with different lengths. The periodicities range between 17 to 100 days. The periodicity ranges are presented in Table 4. The table shows that the periodicity is different for the different types of options and moneyness categories of these options. The periodicity is lowest for DOTM call options, 17 days, and highest for the DOTM puts, OTM and ATM puts, and ITM and ATM calls, 102 days.

TABLE 4 SPECTRAL DENSITY PERIODICITY OF SPY CALL AND PUT OPTIONS IMPLIED VOLATILITY FOR THE PERIOD JANUARY 10, 2005 TO DECEMBER 30, 2005

	Category 1	Category 2	Category 3	Category 4	Category 5
call	68	102	102	51	17
put	102	102	102	51	51

After establishing the periodicity of the SPY option implied volatility wave we can examine the relation between SPY option call and put option implied volatility and the SPY spot volatility. Table 5 presents results for coherence and phase statistics for this relation. The table also reports different kernel specifications for robustness. The coherence statistic, across kernel specifications, is highest and thus rejecting the null hypothesis for the ATM (Category 3) call and put options, coherence in the range – 0.64 to 0.85, which are double than the simple correlation coefficients reported earlier in the paper. The next highest coherences are for the call ITM and put OTM categories - ITM call and OTM put (Category 2), and DITM call and DOTM put (Category 1) options. The coherence ranges are - 0.30 to 0.78, which are again almost double than the simple correlation coefficients reported earlier in the paper. However, keep in mind that correlation coefficients might be wrong because the data are non-normally distributed.

TABLE 5 COHERENCE AND PHASE OF SPY CALL AND PUT OPTIONS IMPLIED VOLATILITY RELATIVE TO THE SPY SPOT STANDARD DEVIATION FOR THE PERIOD JANUARY 10, 2005 TO DECEMBER 30, 2005 AT THE SPECTRAL DENSITY PERIODICITY

	Category 1	Category 2	Category 3	Category 4	Category 5
Bartlett					
coherence					
call	0.30	0.65	0.72	0.17	0.01
put	0.60	0.64	0.64	0.16	0.11
phase					
call	0.94	0.39	0.39	0.38	0.70
put	0.23	0.29	0.26	-3.05	-3.14
Parzen					
coherence					
call	0.32	0.75	0.81	0.24	0.02
put	0.71	0.74	0.75	0.40	0.33
phase					
call	1.03	0.39	0.40	0.22	1.04
put	0.23	0.28	0.26	3.09	3.04
Tukey-Hanning					
coherence					
call	0.32	0.79	0.85	0.33	0.03
put	0.76	0.78	0.80	0.55	0.48
phase					
call	1.12	0.38	0.40	0.18	1.14
put	0.23	0.28	0.26	3.04	3.02

The coherences for the Category 4 and Category 5 options are low, which suggests acceptance of the null hypothesis. These are the OTM and DOTM call options and the ITM and DITM put options. The phase statistics are all positive with the exception of the phase statistics produced by the Bartlett kernel for Category 4 and 5 put options. This suggests that the option implied volatility leads the spot volatility of SPY. Hause (1971) and Stone (1975) suggest that the phase statistic cannot be used to establish lead-lag relations. Nevertheless, one thing is clear the SPY option implied volatility and spot volatility have a lead-lag relation. This also is evidence rejecting the null hypothesis of the study, which suggests the existence of a relation between the ETF spot and ETF option implied volatility.

CONCLUSION

In this study we use spectral analysis on SPY options to examine the relation between spot and implied option volatility for these exchange traded funds (ETF). We attempt to address the question is there a relation between spot and implied volatility or option implied volatility has no relation with the spot ETF volatility. The null hypothesis of the study is spot volatility influencing option implied volatility. To the best of our knowledge this is the first study to address this question. This helps extend our understanding of ETF option behavior. We find that this relation does exist; however, consistently this is true only for the at-the-money call and put options and the in-the-money call and out-of-the-money put categories. Only for these categories of options the coherence statistic is consistently in the range - 0.64 to 0.85.

Using another spectral statistic, the phase statistic, we also find that the SPY option implied volatility and spot volatility have a lead-lag relation - the SPY option implied volatility leads the SPY spot volatility. This is also evidence suggesting that a relation between the ETF spot and ETF option implied volatility exists.

REFERENCES

- Canina, L. and Figlewski, S., 1993, The Informational Content of Implied Volatility. *Review of Financial Studies*, Vol. 6, No. 3, pp. 659-681.
- Christensen, B.J. and Prabhala, N.R. 1998, The Relation between Implied and Realized Volatility. Journal of Financial Economics, Vol. 50, No. 2, pp. 125-150.
- Bakshi, G., Kapadia, N. and Madan, D., 2003, Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options. *Review of Financial Studies*, Vol. 16, No. 1, pp. 101–143.
- Bollen, N. P. B., and Whaley, R.E., 2004, Does Net Buying Pressure Affect the Shape of Implied Volatility Functions? *The Journal of Finance*, Vol. 59, No. 2, pp. 711-753.
- Day, T.E. and Lewis, C.M., 1988, The Behavior of the Volatility Implicit in the Prices of Stock Index Options. *Journal of Financial Economics*, Vol. 22, No. 1, pp. 103-122.
- Hause, J. C., 1971, Spectral Analysis and the Detection of Lead-Lag Relations. *The American Economic Review*, Vol. 61, No. 1, pp. 213-217.
- Ivanov, S. I., Whitworth, J. and Zhang, Y., 2011, The Implied Volatility of ETF and Index Options. *The International Journal of Business and Finance Research*, Vol. 5, No. 4, pp. 35-44.
- Jenkins, G.M., 1965, A Survey of Spectral Analysis. Applied Statistics, Vol. 14, No. 1, pp. 2-32.
- Lakonishok, J., Lee, I., and Poteshman, A.M., 2004, Investor Behavior and the Option Market. NBER Working Paper 10264, http://www.nber.org/papers/w10264.
- Sargent, T. J. (1979) Macroeconomic Theory. Academic Press.
- Stone, M., 1975, Spectral Analysis, Phase Estimates, and Lead-Lag Relationships. *The American Economist*, Vol. 19, No. 1, pp. 52-54.
- Xu, X. and Taylor, S.J., 1994, The Term Structure of Volatility Implied by Foreign Exchange Options. *Journal of Financial and Quantitative Analysis*, Vol. 29, No. 1, pp. 57-74.