Demand-Led Growth, Income Distribution and Debt

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The paper presents a medium-run growth model led by aggregate demand and coherent with minskian financial instability approach. The impact of wage share changes on growth and unemployment is studied within a dynamic framework, characterized by a regime switching technique and a learning mechanism. The model makes two main contributions to the debate: it shows the impact on actual growth, as well as the steady state. Secondly, it illustrates how instability generated by wage changes in a "bad" regime can accelerate the entrance in a "good" regime improving the overall rate of growth without necessarily causing instability in the large.

Keywords: demand-led growth, wage-share, regime switching, learning, simulations, instability

INTRODUCTION

The link between income distribution and economic growth is a strategic topic in the classical political economy since the analysis of Ricardo. From then on, most of the main theoretical paradigms on the economic stage have addressed the issue.

In what follows, attention will be concentrated on a pivotal article by Badhuri and Marglin (1990) and on the ensuing debate. While the two authors have tried "from a broad Keynesian perspective" (ibidem, p.375) to solve the dilemma profit-led versus wage-led growth models, the debate has been mainly carried out within a static Kaleckian framework (see Lavoie, 2014 and Setterfield, 2010 for a review, and Skott, 2016 for a critical position).

These approaches have been called into question along three lines. The first, introduces finance into the model (see Bhaduri and Raghavendra, 2017 and Stockhammer et al., 2019). The second suggests alternative paradigms (Harrodian, as in the case of Palley, 2012, and Skott, 2010; or Kaldorian, as in Ryoo, 2016). Finally, there have been attempts to put the analysis into a dynamic framework (see also Marglin, 2017). Within this strand of literature, demand-led growth models have gained new interest, in particular those based upon the role of autonomous demand. Authors such as Allain (2015), Fazzari, Ferri and Variato (2019) among others, have analysed and deepened different aspects of this model.

The main objective of the paper is to insert the income distribution debate into this strand of literature (see Dutt, 2019 for a comparison between the various approaches). In order to pursue this target, two peculiar innovations are introduced to existing literature.

First of all, the role of autonomous demand is re-discussed. While some authors are strongly against this hypothesis for both theoretical and empirical reasons (see Skott, 2019), in the present paper we temper the hypothesis with the presence of some state variables. At first sight this might sound like an oxymoron. However, it helps overcoming the problem of a fixed rate of growth that would make the fundamental question, wage-led versus profit-led growth, a redundant one. Since in our model, growth is demand-driven, the more correct question must be formulated in terms of growth enhancing versus growth depressing change in the wage share.

The second novelty relates to the choice of analytical tools in order to deal with dynamics. In this paper we will face dynamics using a regime switching technique. The choice of this methodology reveals two comparative advantages when applied to the present topic. On the one hand, it helps reconciling local instability with global stability. A wage share increase may create instability in one regime. However, this process can speed up the entrance into the more "virtuous" one and so improve the overall rate of growth, without compromising global stability. On the other, and this is a consequence of the first aspect, growth can be measured at its historical mean instead of the steady state value and this is particularly important in the case of the present topic.

In addition, the regime switching technique has two further advantages that it is useful to point out. As long as we use an investment function compatible with Minsky's approach, this setting allows us to make a more explicit account of the relevance of his financial instability hypothesis (hereafter FIH, see Ferri, 2019). Secondly, this setting is particularly in keeping with the presence of learning processes. This aspect will be developed in the analysis put forward in the paper, and connects to the recent approach of the so called Heterogeneous Agent Models (hereafter HAMs, see Dieci and He, 2018).

The model is capable of generating a disequilibrium framework where both price and quantities are adjusting, people are learning and markets (goods, labour and financial) generate feedbacks. In this context, wage share changes give rise to three distinct dynamic channels. Not only they have a dual impact as stressed by Bhadhuri and Marglin (1990) who state: "Higher wages mean higher costs of manufacturing, but by providing more purchasing power to the workers they also stimulates demand" (1990, p.375). They may also reduce cash flows and so ignite a debt cycle.

The thesis we put forward is that the presence of three main feedback loops make it difficult to reach solid conclusions: much depends on the values of the parameters used, by the hypotheses made about the specification and by the inclusion of some equations instead of others. However, the methodology followed allows to investigate whether a wage share increase can be growth enhancing without generating instability problems, at least within a non-negligible interval.

The paper outline is the following. The structure of the model we refer to is explained from Section 2 to Section 5. First the role and specification of autonomous demand are explained; then the determinants of income distribution are illustrated. The features of the investment function and its methodological link with minskian ideas are discussed in Section 4; whereas Section 5 presents the conditions for model closure. Sections 6 to 9 carry out different exercises on the basic model. The first step implies comparative statics evaluations. Then the dynamic non-linear model is overviewed. Section 8 illustrates the rationale behind the choice of a regime switching technique. Then Section 9 analyses the impact of the introduction of a learning mechanism. Conclusive remarks and perspectives end the paper as customary. A Mathematical Appendix includes details about the steady state values and the specification of the multipliers used for the linearization of the model.

A TWO-TIER CONSUMPTION FUNCTION WITH AN AUTONOMOUS COMPONENT

The model we are going to discuss is not meant to detail all markets in the economy. We propose a parsimonious pedagogical structure where the explicit equations are introduced only in order to highlight the variables under study and their contribution to macroeconomic dynamics. The model is self-contained and faces demand and supply equations, covering both the product and the labor market. Goods market is specified so to show up the interdependence with financial conditions. Furthermore equations looking like Taylor rule and Phillips curve enable to establish links between nominal and real variables, along with policy variables and expectations. All equations are expressed (except the rate of interests) in intensive form, that is deflated by last period output or nominal output when the variable under study is defined in nominal terms. The model considers the interaction of two functional sectors: households and firms. Hence the distribution of income refers just to the two categories of wages and profits. Nevertheless, the particular structure of the model makes clear that there is no coincidence between wage share of income and households share of income. There is no further explicit role for the State except through the setting of the monetary rule.

The model refers to a medium-run period, defined by Minsky (1982), "intermediate" and capable of covering a full cycle experience, but not long enough to take changes in human capital, institutions or ecological aspects into account.

The environment where the pattern of wage share is studied is derived from models characterized by an aggregate demand driver. We have explored this route many times, since the initial paper by Fazzari, Ferri and Variato (2019) (see also Ferri, 2019 and Ferri, Cristini and Variato, 2019).

Two characteristics of this approach are to be considered in a preliminary way. First of all the driver of the model is portrayed by some autonomous component of aggregate demand. Let's refer to durable consumption (F_t), which can be represented in the following way:

$$F_t = F_{t-1}(1+g^*) \tag{1}$$

Then suppose autonomous consumption grows at the exogenous rate g*, which is the driver of the system. In other words, durable consumption does not depend on the state of the economy.

Now move to an intensive form representation (i.e. divided by last period output Y_{t-1}). This leads to:

$$f_t = \frac{f_{t-1}}{(1+g_{t-1})}(1+g^*) \tag{2}$$

This assumption has been criticized (see Skott, 2019) on both theoretical and empirical grounds. The following specification is put forward, in order to mitigate the critique:

$$f_t = \frac{f_{t-1}}{(1+g_{t-1})}[1+g^*(1-\kappa_1 u_{t-1})] \tag{3}$$

In this case, demand for durable goods depends on an exogenous rate of growth g* as before, but this rate is moderated by the presence of unemployment.

The analytical implications of this simple change are however pervasive: the rate of growth is no longer exogenous, even though the model remains demand-driven.

The second aspect to be emphasized relates to the structure of the consumption function we are going to use. We suggest a two-tier consumption function where the durable component is completed by the non-durable component as expressed in the following equation:

$$c_t = (1+Eg_t)(c_1 - c_2 u_{t-1}) + c_3 R_t d_t \tag{4}$$

Such specification makes the non-durable demand for goods depend upon three factors: expected growth, unemployment and financial inflows. Though these three components are mostly introduced in a simple linear way, and given the lag structure, we can see unemployment exerts a threefold action. It increases uncertainty, then directly reduces consumption; it negatively affects expectations (see Ferri, Cristini and Variato, 2019). These explicit effects are magnified by the implicit impact on the overall propensity to consume, given the heterogeneity between employed and unemployed workers. This seed of heterogeneity is reinforced by the presence of financial creditors who spend c_3 of their financial income, represented by $R_t d_t$. In fact, the impact of debt for borrowers, shown in the investment function, is different (as it will become evident in section 4).

The expected rate of growth is defined through a simple adaptive rule where the lag structure is at most of order one. It comes as a weighted average of two components: one period lagged expected rate of growth and one period lagged actual growth.[1]

$$Eg_t = (1-\alpha)g_{t-1} + \alpha Eg_{t-1} \tag{5}$$

INCOME DISTRIBUTION

Prices are now introduced into the analysis (see Ferri, 2019), differently from what has been done in Fazzari, Ferri and Variato (2019). This implies that two kinds of adjustments coexist in the system: one based upon prices and the other on quantities.

In this context, the pricing equation, normally named the Phillips curve[2] is represented by the following expression:

$$\pi_t = E\pi_t - \psi_1(u_{t-1} - u_0) \tag{6}$$

where $E\pi_t$ represents expected inflation, while the subscript 0 stands for steady state values. In this case too, we use the same type of structure adopted for the previous expected value:

$$E\pi_t = (1-\beta)\pi_{t-1} + \beta E\pi_{t-1} \tag{7}$$

This equation is not necessarily micro-founded, while expectations can be specified in different ways, ranging from bounded rationality to some process of learning (see Ferri, Cristini and Variato, 2019). The presence of lagged unemployment is justified by the need to generate a recursive system. However, it has also an empirical support. Later on, when deriving a compact linearized model, this assumption will be dropped.

The above equation is the first step in the direction of obtaining income distribution. To this purpose, we can take a short cut and assume that the wage share (ω_t) is a function of the type:

$$\omega_t = \omega_{t-1}[\,1\text{-}\xi_1(g_{t-1} - g_0)] \tag{8}$$

where g_t is the rate of growth.

In the present model, productivity is expressed by the following Kaldor-Verdoorn equation:

$$\tau_t = \theta_0 + \theta_1(\tfrac{i_{t-1}}{v^*} - \delta) \tag{9}$$

where i_t is investment, v* is the desired capital-output ratio and δ is the depreciation rate. It follows that the wage growth equation is implicitly derivable from the above three equations.

Two kinds of observations are important at this stage of the analysis. The first is that one can study the dynamic impact of income distribution by considering both changes in the parameter ξ_1 and the value of the exogenous steady state value (ω_0).[3] The second is that the above specification (8) implies that for a positive ξ_1 labor share moves counter-cyclically. However, differently from asserted by Bhaduri and Marglin (1990, p.379) who refer to a static model, this assumption does not necessarily imply the existence of a profit led model in the present context.

A MINSKIAN INVESTMENT FUNCTION AND THE PROCESS OF ACCUMULATION

The investment function is based upon a simplified Minskian function (see Fazzari, Ferri and Greenberg, 2008). It ignores the presence of a triptych of agents characterized by a different financial position (see Davis et alia, 2019). However, at this stage of the analysis, where the financial aspects are elementary sketched, the specification seems functional to capture the essence of FIH either in a long-run perspective or in the presence of a run-away situation.[4]

The following specification is rich of implications, as it shows an eclectic nature where explicit reference is given to the roles of expected growth, income distribution, external financing and related outflows, real interest rate and capacity utilization:

$$i_t = \eta_1(1 + Eg_t) + \eta_2(1 - \omega_t) - \eta_3 R_t d_t - \eta_4(r_t - r_0) - \eta_5(1 - h_t) \qquad (10)$$

As in standard notation, r_t is the real rate of interest (where the subscript 0 stands for the steady state value), while h_t represents capacity utilization. This specification make real and financial aspects interdependent and implies both quantity and price adjustments.[5] Furthermore, it separates the role of gross profit share (η_2) from the impact of interests on existing debt (η_3). We will show how these characteristics play a fundamental role in shaping the dynamics.

Capacity utilization is represented by:

$$h_t = h_{t-1}\frac{1+g_t}{1+g_t^k} \qquad (11)$$

The rate of growth of capital (g^k) is given by:

$$g_t^k = \frac{i_{t-1}}{v_{t-1}} \qquad (12)$$

while the capital-output ratio is equal to:

$$v_t = \frac{v_{t-1}}{(1+g_{t-1})}(1 - \delta) + \frac{i_{t-1}}{(1+g_{t-1})} \qquad (13)$$

This equation represents the impact of investment on the growth of capacity.

CLOSING THE MODEL

The remaining variables included in (10), i.e. R_t and d_t are now going to be specified.

$$R_t = R^* + \gamma_1(E\pi_t - \pi^*) - \gamma_2(u_{t-1} - u_0) \qquad (14)$$

The above expression is the equation which determines the nominal rate of interest according to a Taylor rule, where π^* is a exogenous target of inflation. Then, from (14) and from (6), one derives the equation of the real rate of interest:

$$r_t = \frac{(1+R_t)}{(1+\pi^*)} - 1 \qquad (15)$$

The (business) debt equation is defined in the following way:

$$d_t = \frac{d_{t-1}}{(1+g_{t-1})(1+\pi_{t-1})}(1 + R_t) + \frac{i_{t-1}}{(1+g_{t-1})} - (1 - \omega_t) \qquad (16)$$

The debt of firms increases in order to face the outflows of new investments and interest rates, whereas is decreased by the availability of profit flows; all figures are expressed in intensive form. By comparing (10) and (16), one can identify the two roles of the profit share: as a stimulus to investment and as a source of finance. This dual aspect enriches the dynamics.

Finally, the following equations close the model.

$$g_t = f_t + i_t + c_t - 1 \tag{17}$$

$$gN_t^s = \rho_0 - \rho_1 u_{t-1} \tag{18}$$

$$gN_t = \frac{(1+g_t)}{(1+\tau_t)} - 1 \tag{19}$$

$$u_t = 1 - (1 - u_{t-1})\frac{(1+gN_t)}{(1+gN_t^S)} \tag{20}$$

where (17) represents the equilibrium on the product market, while the remaining equations belong to the labour market. Labour supply is endogenous (18), while (19) stands for the demand for labour.

A COMPARATIVE STATICS EXERCISE

Given an exogenous rate of autonomous demand growth g* and price expectation π*, the system refers to 15 unknowns: p_t, R_t, c_t, i_t, f_t, g_t, τ_t, gN_t^S, gN_t, u_t, d_t, w_t, Eg_t and $E\pi_t$ inserted into 15 equations.

In the present paper the consideration of steady state is particularly important because in trying to discover the impact of wage share changes, we shall start from these values. The steady state values (marked by the subscript 0) relative to growth, unemployment and debt are worth considering.

From (3), the steady state rate of income growth is given by the following expression:

$$g_{0=}[1 + g^*(1 - \kappa_1 u_0)] \tag{21}$$

where one can better understand the meaning of an exogenous driver (g*) tempered by the presence of unemployment.

The debt ratio is given by:

$$d_0 = \frac{i_0 - \mu_1(1-\omega_0)(1+g_0)}{\Delta} \tag{22}$$

where

$$\Delta = g_0 - \mathrm{r}_0 (23)$$

is a discount factor, assumed to be constant.[6]

The steady state rate of unemployment is given by:

$$u_0 = \frac{[\rho_0 + \tau_0(1+\rho_0)] - g_0}{\rho_1(1+\tau_0)} \tag{24}$$

It is important to underline this value does not represent a natural rate.

According to Solow (2018), there is no well-defined natural rate of unemployment, either statistically or conceptually" (p.423). The above specification is in keeping with this statement.[7]

Let us consider the parameters shown in Table 1.

TABLE 1
THE VALUES OF THE PARAMETERS FOR THE NONLINEAR MODEL

Relevant reference functions	Benchmark values
Consumption	c_1=0.60 c_2 =0.25 c_3=0.25
Investment	η_1 =0.28 η_2 =0.295 η_3=0.50 η_4 =0.05 η_5=0.10
Taylor rule	γ_1=1.5 γ_2 =0.20
Phillips Curve	ψ_1=0.10
Speed of expectations adjustment	a=b=0,5
Growth rate of Autonomous Demand	g*=0.04
Income distribution	ξ_1 =0.10
Productivity (and depreciation rate)	θ_0 =0.02 θ_1=0.10 δ=0.10
Labor Supply	ρ_0 =0.03 ρ_1 =0.35
Price Expectation	π*=0.031
Target Interest Rate	R*=0.035
Wage Share	ω*=0.75

In this context, an increase in ω^*, for example from 0.75 to 0.80, leaves growth and unemployment unchanged. Only a switch from investment to consumption takes place, while debt increases. In other words, the impact seems to be more on the financial variables than on the real ones.

In this perspective, supposed to set η_3=0 (this is the parameter linking investment to the interest share). The results are the same as before, except that the increase in debt is much stronger.

In order to understand the different dynamic impact of these changes, we are now ready to leave the comparative statics territory.

A DYNAMIC OVERVIEW OF THE NONLINEAR MODEL

The nonlinear system illustrated before and characterized by the parameters of Table 1, can generate the following dynamics illustrated in Figure 1.

FIGURE 1
DYNAMICS: GROWTH, UNEMPLOYMENT, DEBT AND WAGE SHARE

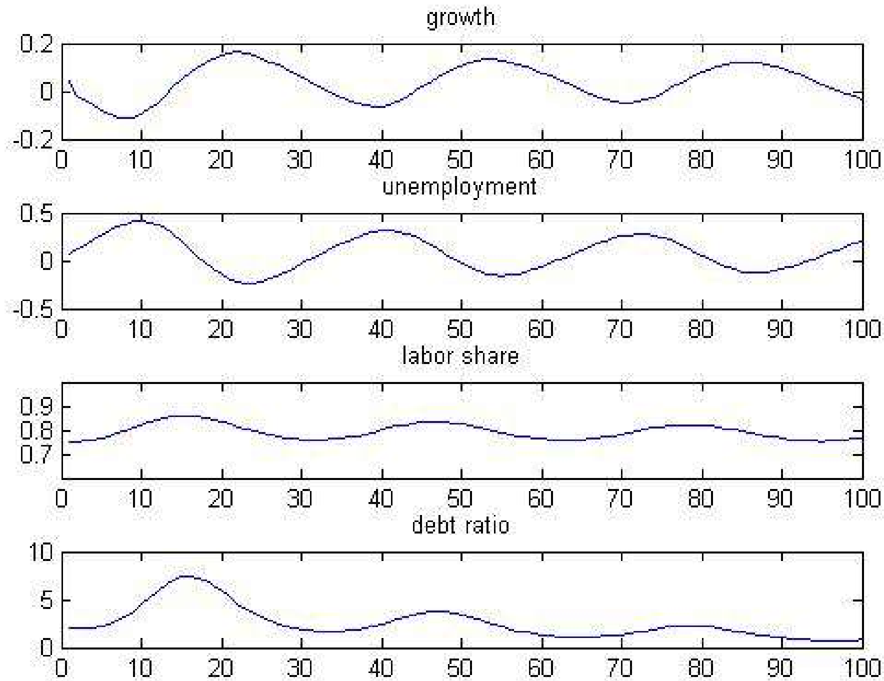


Figure 1 shows endogenously generated fluctuations in growth, the rate of unemployment, the wage share and the (business) debt ratio.

Obviously, the results are not relevant for their numerical value: what matters most is the qualitative dynamics we observe even starting from a quite basic specification. Some considerations can be put forward. Primarily the model generates persistent and bounded fluctuations in a medium-run perspective. Secondly, the process of growth is accompanied by the presence of a positive rate of unemployment. Thirdly, there is a discrepancy between the peaks and the trough respectively of g_t and ω_t. Finally, there is not a significant correlation between debt and growth, even though the debt ratio rises before the boom: this pattern is coherent with the theoretical implications of Minsky's financial instability hypothesis.[8]

If one linearizes the model (see the Mathematical Appendix), one discovers that when $\omega_0 = 0.75$ the modulus of the system is equal to 1 so that the economy fluctuates. However, when ω_0 is increased, the modulus decreases to 0.9875. This means that fluctuations continue but the system tends to stabilize in the long-run.

It follows that an increase in ω_0, although neutral in real terms, can contribute to stabilize the system. On the contrary, when $\eta_3 = 0$ (i.e. investment is no longer sensitive to interest repayments), the system tend to explode and an increase in ω_0 make the situation even worse. These results stress the importance of the interrelations between real and financial aspects.

INTRODUCING REGIME SWITCHING

In order to deepen the dynamic aspects, a regime switching technique is introduced (see Ferri, 2011). This method is based upon three strategic elements:

a) there are multiple equilibria (two, at least);
b) there is a threshold dividing them;
c) some equations switch in the various states.

In the present case, different thresholds will be considered. In fact, they can either be deterministic or stochastic. We adopt a framework with two regimes. We will refer to Regime I as the "bad" regime, and Regime II as the "good" regime. Then we start from the following deterministic threshold given by the following inequality:

$$g_{t-1} > \frac{g_{01}+g_{02}}{2} \quad (25)$$

The left hand side represents last period rate of growth. If this number exceeds the threshold, which is formed by the mean of the two steady rates of growth, Regime II, the "virtuous" one, is prevailing. The opposite happens in Regime I, the "vicious" one.

There are two fundamental switches in the system. The first is the value of the exogenous rate of growth of autonomous demand, g*, which therefore becomes g_j^*; here j=1,2 indicates the prevailing regime. The second kind of switch takes place in the parameters of the various equations, as indicated in Table 2.

TABLE 2
THE VALUES OF THE PARAMETERS IN THE REGIME-SWITCHING MODEL

Relevant function	Parameters and regime values
Consumption	c_1=0.58 c_{21}=0.15 c_{22}=0.10 c_3=0.25
Investment	η_{11}=0.29 η_{12} =0.31 η_{21}=0.30 η_{22} =0.30 η_{31}=0.50 η_{32}=0.40 η_4=0.5 η_5=0.05
Productivity (and depreciation rate)	θ_0 =0.03 θ_{11}=0.10 θ_{12}=0.05 δ=0.10
Autonomous demand and u	$\kappa_{11} = 0.6$ $\kappa_{12} = 0.4$
Unemployment	ρ_0 =0.03 ρ_1 =0.65
Income distribution	ξ_1=0.10
Growth rate of autonomous demand	g^*_{01}=0.02 g^*_{02}=0.04
Speed of expectations adjustment	a=0.80 β=0.80
Phillips Curve	$\psi_{11} = 0.05$ $\psi_{12} = 0.03$
Taylor Rule	γ_{11}=1.2 γ_{12}=1.8 γ_{21}=0.4 γ_{22}=0.2

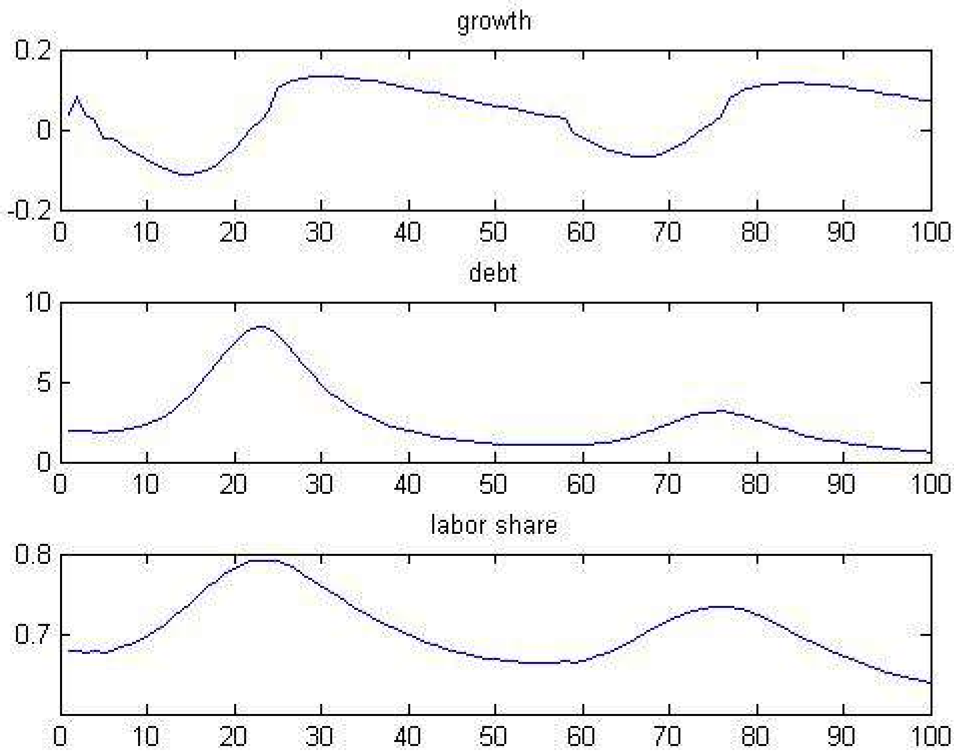
These parameters determine the different values of the steady states illustrated in Table 3 and of course influence the dynamics of the model.

Regime I is the "bad" regime where growth is low and the debt ratio is substantial. Regime II has the opposite characteristics. However, they have the same dynamic properties.[10] The dynamics of the whole model are shown in Figure 2.

TABLE 3
THE STEADY STATE VALUES IN THE 2 REGIMES

	Regime I	Regime II
g_0	0.0192	0.0394
d_0	4.87	3.14
f_0	0.0795	0.0595
c_0	0.5873	0.6268
i_0	0.3524	0.3542
ω_0	0.7	0.68
u_0	0.0651	0.0350
π_0	0.0101	0.0250

FIGURE 2
THE DYNAMICS OF THE META-MODEL



The following aspects are worth stressing. The first is that also this the model is capable of generating bounded fluctuations. However, these fluctuations are more volatile than those considered so far. This means that the model can deal with important financial crises having big impact on real variables. Finally, the system is robust in the sense that changes in the parameters maintain the properties of the model.

A particular kind of sensitivity analysis is important for our purposes and this refers to the change in income distribution. At present the wage share is 0.70 for Regime I and e 0.68 for Regime II.

With these values along with those shown in Table 2, the average rate of growth is 0.0381. Different combinations of steady states generate different actual growth rates and possibly a variety of dynamic patterns.

TABLE 4
THE IMPACT OF CHANGES IN ω_0

Deterministic		Average g_t	
$\omega_{01} = 0.7$	$\omega_{02} = 0.685$	0.0381	Stable
$\omega_{01} = 0.77$	$\omega_{02} = 0.75$	0.0405	Stable
Stochastic		Average g_t	
$\omega_{01} = 0.74$	$\omega_{02} = 0.70$	0.0199	Stable
$\omega_{01} = 0.76$	$\omega_{02} = 0.72$	0.0191	Stable

This holds true for both deterministic and stochastic threshold, where a random variable normally distributed (0, 0.05) is appended.[10]

In order to understand the results, one has to distinguish between stability of the regime vis-à-vis the global stability of the overall system. The fact that Regime I is unstable and that an increase in ω_{01} maintain this property is not a negative property for the simple fact that it leads the system into the other regime in a speedier way. The overall system may remain stable, if that trespassing is not overshooting,[11] while the average rate of growth is increasing. It follows that a wage share increase can be both growth-enhancing and stabilizing.

LEARNING WITH HETEROGENEOUS BELIEFS

It is rather important at this stage of the analysis to challenge the assumptions made about both growth and price expectations. The adaptive formula that has been used has been strongly criticized by the rational expectations supporters.

It must be stressed that, from an empirical point of view, the adaptive formula for prices seems to mimic reasonably well the survey of Professional forecasters, while the same is not true for growth expectations, where the signs of the Great Recession have been stronger. In this case, it is convenient to refer to some form of learning.

The number of learning devices is really numerous so that one cannot pretend to be exhaustive. However, in what follows, a particular learning technique will be adopted. Let us assume that agents do not have a complete knowledge of the model and therefore they use simple rules to forecast the future output growth and inflation. We suppose, as is done in De Grauwe (2008), that the agents can be either optimistic or pessimistic (see also Ferri, Cristini and Variato, 2019):

The optimists forecast is given by the following relationship:

$$\overset{-}{E}{}_t^{opt}\, g_{t+1} = g_{2,t}^* \tag{26}$$

In other words they expect that a bigger rate of growth is always prevailing. On the other hand, the pessimists forecast a smaller rate, which can even be negative:

$$\overset{-}{E}{}_t^{pess}\, g_{t+1} = g_{1,t}^* \tag{27}$$

The market forecast is obtained as a weighted average of these two forecast, i.e.:

$$\alpha_{opt,t}\overset{-}{E}{}_t^{opt}\, g_{t+1} + \alpha_{pess,t}\overset{-}{E}{}_t^{pess}\, g_{t+1} \tag{28}$$

$$\alpha_{opt,t} + \alpha_{pess,t} = 1 \tag{29}$$

Following Brock and Hommes (1997), a selection mechanism is introduced. In fact, agents compute the forecast performance by referring to the mean squared forecasting error:

$$U_{opt,t} = -\sum_{k=1}^{\infty} \chi_k \left[g_{t-k} - \bar{E}_{opt,t-k-1} g_{t-k}\right]^2 \quad (30)$$

$$U_{pess,t} = -\sum_{k=1}^{\infty} \chi_k \left[g_{t-k} - \bar{E}_{pess,t-k-1} g_{t-k}\right]^2 \quad (31)$$

where χ represents geometrically declining weights.

TABLE 5
BENCHMARK VALUES OF THE PARAMETERS IN THE CASE OF LEARNING

Reference functions	Benchmark Parameters
Consumption	c_1=0.60 c_{21}=0.25 c_{22}=0.15 c_3=0.25
Investment	η_{11}=0.29 η_{12} =0.31 η_{21}=0.27 η_{22} =0.30 η_{31}=0.40 η_{32}=0.40 η_4=0.5 η_5=0.01
Taylor rule	γ_{11}=1.2 γ_{12}=1.8 γ_{21}=0.6 γ_{22}=0.2
Phillips curve	$\psi_{11} = 0.11$ $\psi_{12} = 0.07$
Growth rate Autonomous Demand	g_{01}*=0.015 g^*_{02} = 0.035
Autonomous demand and u	$\kappa_{11} = 0.6$ $\kappa_{12} = 0.4$
Expectation speed of adjustment (just inflation)	β=0.95
Labor Supply	ρ_0=0.03 ρ_1=0.35
Income Distribution	ξ_1=0.10
Productivity	θ_0=0.0042 θ_{11}=0.15 θ_{12}=0.10 δ=0.10
For learning:	γ=10.000 ρ=0.2

The proportion of agents are determined à la Brock and Hommes (1997):

$$\alpha_{opt,t} = \frac{exp(\gamma U_{opt,t})}{exp(\gamma U_{opt,t}) + exp(\gamma U_{pess,t})} \quad (32)$$

This learning mechanism has been inserted into the system illustrated in the previous Section. Furthermore, the threshold has been modified in the following way:

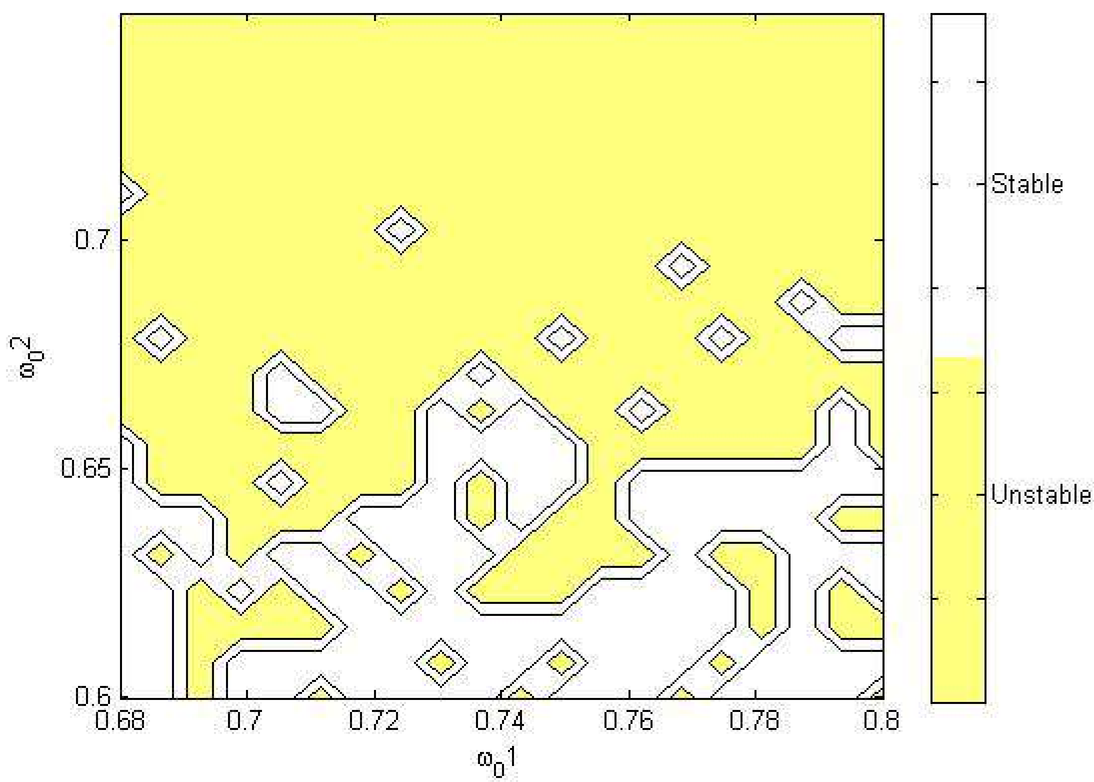
$$g_{t-1} > \frac{g_{01} + g_{02}}{2} + \epsilon_t \quad (33)$$

where the last term represents a stochastic variable, normally distributed with 0 mean and σ=0.05. The values of the parameters for the new Monte Carlo simulations, repeated 100 times, are presented in Table 5.

The values of the ω_{0j} have not been indicated because we allow for some changes in a range of values as shown in Figure 3.

This final experiment allows us to point out some interesting result. Through the introduction of learning, the two questions are unified referring respectively to the impact on growth and the nature of the dynamic process. Secondly the presence of a regime switching technique along with the hypothesis of learning have produced a much greater interval of stability. Nevertheless, such endogenous stabilizing mechanisms have not eliminated the possibility of instability which turns out to depend on both the values of the parameters chosen and on the specification put forward.

FIGURE 3
THE DYNAMIC IMPACT OF CHANGES IN THE WAGE SHARE



CONCLUDING REMARKS

The paper has presented a medium-run model of growth led by aggregate demand and characterized by a simplified minskian investment function, where real and financial aspects coexist and demand and supply interacts. Furthermore, unemployment is a structural property of the model where the wage and price dynamics take place along a moderate path.

In this context the impact of changes in income distribution, distinguished between changes in the steady state value and changes in the parameters affecting its dynamics, has been studied in successive steps, keeping in mind that change in wage share not only impacts on aggregate demand or on cost structure but also on cash flows and hence on debt.

Initially, the problem has been faced in two steps. The comparative statics analysis was meant to discover the impact on steady state growth, while stability analysis was supposed to ascertain the dynamic properties. In this double perspective, the results are not very interesting because, given the assumption of an exogenous autonomous demand, although modified, the rate of growth does not change, while the dynamics tend to be destabilizing.

A different picture emerges if in one introduces a regime switching technique supplemented by a learning mechanism. According to Dieci and He (2018) this is a case of a heterogeneous agent model (HAMs), which reinforces the other elements of heterogeneity implicit in the two-tier consumption function adopted along with the behavioural differences between debtors and creditors.

The model makes to main contributions to the debate. On one hand, it measures the impact on actual growth and not only on a steady state concept. On the other, it shows how instability generated by the wage change in a "bad" regime" can speed up the entrance in the "good regime" so that the overall rate of growth is improved without necessarily causing instability in the large.

These properties, however, have not eliminated the possibility of instability that depends not only on the values of the parameters chosen but also on the specification put forward and the omissions which have been tolerated. Two in particular are worth mentioning that constitute an area of future research. On the one hand, it would be more meaningful to discuss the causes underlying the increase or decrease in the wage share. For instance, the consideration of a rising corporate market power would make the story much more telling (see Akcigit and Ates, 2019 and IMF, 2019). On the other, the impact of income distribution changes on supply should be deepened.

ENDNOTES

1. All expectation operators (E) will be presented without the subscript for notational convenience, as they refer to the same period of the actual variable.
2. This is a misnomer because the Phillips curve is an empirical object.
3. The hypothesis that this value is endogenous will also be dealt with.
4. Ferri (2013) has introduced this richer structure in the consumption function.
5. For the difference between this specification based upon financial instability hypothesis and the so called financial accelerator model, see Stockhammer et al. (2019).
6. Note that this relationship is the opposite that one meets for capital in an intertemporal model. See Barro (2019).
7. The Mathematical Appendix shows how multiple equilibria can be present and why they can be disregarded.
8. On this empirical aspect, see Lavoie (2014), Ferri (2019) and Stockhammer et al. (2019).
9. When linearized, they both show the modulus of the maximum eigenvalue to be >1.
10. In this case, Monte Carlo simulations, repeated 100 times, have been carried out.
11. In other words, it cannot go beyond the value of the steady state in Regime II, which is also unstable. In this case, a runaway situation is generated.

REFERENCES

Allain, O. (2015). Tackling instability of growth: A Kaleckian model with autonomous demand expenditures, *Cambridge Journal of Economics*, 39(5), 1351-1371.

Akcigit, U., & Ates, S.T. (2019). Ten facts on declining business dynamism and lessons from endogenous growth theories. *NBER Working Papers*, 25755. Cambridge (MA).

Barro, R. J. (2019). Double-counting of Investment. *NBER Working Papers*, 25826. Cambridge (MA).

Bhaduri, A., &Marglin, S. (1990). Unemployment and the real wage: the economic basis for contesting political ideologies, *Cambridge Journal of Economics*, 14(4), 375-393.

Bhaduri, A., & Raghavendra, S. (2017). Wage and profit led-regimes under modern finance: an exploration, *Review of Keynesian Economics*, 5(3), 426-438.

Brock, W. A., & Hommes, C. (1997). A rational route to randomness. *Econometrica*, 65(5), 1059-1095.

Davies, L. E., De Souza, J. P. A., & Hernandez, G. (2019). An empirical analysis of Minsky regimes in the US economy. *Cambridge Journal of Economics*, 43(3), 541-584.

De Grauwe, P. (2008). DSGE-Modelling: when agents are imperfectly informed. *European Central Bank Working Paper*, n. 897.
Dieci, R., & He, X.Z., (2018). Heterogeneous agent models in finance. In *Handbook of Computational Economics*, (volume 4, pp. 258-320). Amsterdam: North-Holland.
Dutt, A. K. (2019). Some observations on models of growth and distribution with autonomous demand growth. *Metroeconomica*, 70(2), 288-301.
Fazzari S., Ferri P., & Greenberg, E. (2008). Cash flow, investment and the Keynes-Minsky Cycles. *Journal of Economic Behavior and Organization*, 65(3-4), 555-572.
Fazzari, S., Ferri, P., & Variato, A.M. (2019), Demand-led growth and accommodating supply. *Cambridge Journal of Economics*, forthcoming. https://doi:.org/10.1093/cje/bez055, first presented (2018) as FMM, Working Paper n. 15, IMK Macroeconomic Policy Institute.
Ferri, P. (2011). *Macroeconomics of Growth Cycles and Financial Instability*. Cheltenham: Edward Elgar.
Ferri, P. (2013). Income distribution and debt in a fragile economy: Market processes and macro constraints. *Journal of Economic Interaction and Coordination*, 8, 219-230.
Ferri, P. (2019). *Minsky's Moment: an insider view on the economics of Hyman Minsky*. Cheltenham: Edward Elgar.
Ferri, P., Cristini, A., & Variato, A. M. (2019), Growth, unemployment and heterogeneity. *Journal of Economic Interaction and Coordination*, 14(3), 573-593.
IMF. (2019, April). *The rise of corporate market power and the macroeconomic effects*. W.E.O, Washington.
Kuznetsov, Y. A. (2004). *Elements of Applied Bifurcation Theory*. New York: Springer-Verlag.
Lavoie, M. (2014). *Post Keynesian Economics: New Foundations*. Cheltenham: Edward Edgar.
Marglin, S. A. (2017). Wages, prices and employment in a Keynesian long run. *Review of Keynesian Economics*, 5(3), 360-425.
Minsky, H. P. (1982). *Can "it" happen again?* New York: Sharpe.
Palley, T.I. (2012). Growth, unemployment and endogenous technical progress: A Hicksian resolution of the Harrod's knife edge. *Metroeconomica*, 63(3), 512-41.
Ryoo, S. (2016). Inequality of income and wealth in the long-run: a kaldorian approach. *Metroeconomica*, 67(2), 429-457.
Setterfield, M. (ed.) (2010). *Handbook of Alternative Theories of Economic Growth*. Northampton: Edward Elgar
Skott, P. (2010). Growth, instability, and cycles: Harrodian and Kaleckian models of accumulation and income distribution. In M. Setterfield (Ed.), *Handbook of Alternative Theories of Economic Growth* (pp. 108-131). Northampton: Edward Elgar.
Skott P. (2016). Weaknesses of 'wage-led' growth. *Working Paper*, 2016-08. Amherst, University of Massachusetts.
Skott, P. (2019). Autonomous demand, Harrodian instability and the supply side. *Metroeconomica*, 70(2), 233-246.
Solow, R. (2018). A theory is a sometime thing. *Review of Keynesian Economics*, 6(4), 421-424. NBER.
Stockhammer, E., Jump, R.C., Kohler, K., & Cavallero, J. (2019). Short and medium term financial-real cycles: an empirical assessment. *Journal of International Money and Finance*, 94, 81-96.

APPENDIX

A) STEADY STATE

We only indicate two steady states. Recalling the previous equations numbers, one is unemployment which is equal to:

$$u_0 = \frac{[\rho_0 + \tau_0(1+\rho_0)] - g_0}{\rho_1(1+\tau_0)} \tag{24}$$

where $g_0 = g^*[1-\kappa_1 u_0]$.

The other is the steady state debt ratio:

$$d_0 = \frac{\frac{i_0-(1-\omega_0)(1+g_0)}{\Delta}}{} \tag{22}$$

where $\Delta = g_0 - \mathrm{r}_0$ (23)

By means of a process of substitution, one obtains an equation of the second order for u_0. This does not lead to multiple equilibria because one root is greater than 1.

B) COMPARATIVE STATICS

As before, call $\Delta = g_0 - \mathrm{r}_0$, then consider the following definitions:

$$CSC1 = c_1 - c_2 u_1$$
$$CSC2 = c_2(1+g_1)$$
$$CSD1 = \omega_1/\delta$$
$$CSD2 = \frac{1}{\delta} + \frac{g_1}{\delta}$$
$$CSUD = (\rho_1 + \rho_1\tau_1)^2$$
$$CSU1 = \frac{\theta_1[(\rho_1+\rho_1\tau_1)(1+\rho_0)]}{CSUD} - \frac{\theta_1[\tau_1(1+\rho_0)\rho_1 + g_1\rho_1]}{CSUD}$$
$$CSU2 = \frac{1}{\rho_1(1+\tau_1)}$$
$$CSU3 = CSU1 - CSU2$$

$$\begin{bmatrix} 1 & 0 & 0 & \eta_3 R_0 & -\eta_1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -c_3 R_0 & -CSC1 & CSC2 \\ 1/\Delta & 0 & 0 & 1 & -CSD1 & 0 \\ 0 & 0 & 0 & 0 & 1 & g_0\kappa_1 \\ 0 & 0 & 0 & 0 & -CSU3 & 1 \end{bmatrix} \begin{bmatrix} di_0 \\ df_0 \\ dc_0 \\ dd_0 \\ dg_0 \\ du_0 \end{bmatrix} = \begin{bmatrix} -0.05\eta_2 \\ 0 \\ 0 \\ 0.05CSD2 \\ 0 \\ 0 \end{bmatrix}$$

C) LINEARIZING THE MODEL

In order to continue our investigation on the dynamic impact of changes in the wage share , it is convenient to linearize the model around its steady state values. Furthermore, the dimension of the system is reduced to 10-D, having substituted the equations (4), (6),(9),(12),(14), (15), (18) and (19) into the system.

The system can be written in the following compact 10-D way, where all the variables represents deviations from steady state values.

$$Eg_t = \alpha g_{t-1} + (1-\alpha) g_{t-1} \quad (1a)$$

$$E\pi_t = E\pi_{t-1} - E1u_{t-1} \quad (2a)$$

$$\omega_t = \omega_{t-1} - O1g_{t-1} \quad (3a)$$

$$d_t = D1d_{t-1} + D2E\pi_t - D3u_t + D4u_{t-1} - D3g_{t-1} - D4E\pi_{t-1} + D5i_{t-1} + \omega_t \quad (4a)$$

$$h_t = h_{t-1} + H1g_t - H2i_{t-1} + H3v_{t-1} \quad (5a)$$

$$f_t = f_{t-1} - F1g_{t-1} - F2u_{t-1} \quad (6a)$$

$$g_t = -C1u_{t-1} + C2Eg_t + C2E\pi_t - C3u_t + C4d_t + i_t + f_t \quad (7a)$$

$$v_t = V1v_{t-1} - V2g_{t-1} + V3i_{t-1} \quad (8a)$$

$$i_t = \eta_1 Eg_t - I1E\pi_t - \eta_2\omega_t + I2u_t - I3d_t + \eta_5 h_t \quad (9a)$$

$$u_t = U1u_{t-1} - U2g_t + U3i_{t-1} \quad (10a)$$

The system can be put in a compact matrix form.

$Ax_t = Bx_{t-1}$ or $x_t = A^{-1}Bx_{t-1}$

Some eigenvalues of this system are complex conjugate and have unitary modulus, named $|\lambda|=1$, while the remaining are less than 1. Thus the fixed point has a bifurcation analogous to the Neimark bifurcation, with limit cycles being generated (see Kuznetsov, 2004, for the study of necessary and sufficient conditions).

We want to investigate the impact of a wage share increase, by using substantially the same parameters of Table 1. It must be stated that small changes with respect to the previous simulations have been introduced since some lags that were justified in order to obtain a recursive model (such as in the equation (6)) have been dropped.

In equation (15), the multipliers are:

$$R1 = \frac{1}{(1+\pi_0)}$$

$$R2 = \frac{(1+R_0)}{(1+\pi_0)^2}$$

In equation (2a), $E1 = (1+b)y_1$

For equation (8), one has: $O1 = \xi_1\omega_0$

In equation (16):

$$D1 = \frac{(1+R_0)}{(1+\pi_0)(1+g_0)}$$

$$D2 = \frac{d_0}{(1+\pi_0)(1+g_0)}$$
$$D3 = \frac{d_0(1+R_0)}{(1+\pi_0)^2(1+g_0)} + \frac{i_0}{(1+g_0)^2}$$
$$D4 = \frac{d_0(1+R_0)}{(1+\pi_0)^2(1+g_0)}$$
$$D5 = \frac{1}{(1+g_0)}$$

In equation (4), one has:

$C1 = (1+g_0)c_2$
$C2 = (c_1 - c_2 u_0)$
$C3 = c_3 d_0$
$C4 = c_3 R_0$

In equation (10):

$I1 = I2 = \eta_3 d_0$
$I3 = \eta_3 R_0$

For equation (3), one has: $F1 = \frac{f_0}{(1+g_0)}$

In equation (19),

$GN1 = 1/(1+\tau_1)$
$GN2 = (1+g_0\,)/(1+\tau_0)(1+\tau_0)$

For equation (20), one has:

$$U1 = \frac{1+g_0}{1+g_{Ns,0}}$$
$$U2 = \frac{1-u_0}{1+g_{Ns,0}}$$
$$U3 = \frac{(1-u_0)(1+g_0)}{(1+g_{Ns,0})}$$

For (13):

$$V1 = \frac{1-\delta}{1+g_0}$$
$$V2 = \frac{(1-\delta)v_0 - i_0}{(1+g_0)^2}$$
$$V3 = \frac{1}{1+g_0}$$

For (12), $K1 = \frac{1}{v_0}$ and $K2 = \frac{i_0}{v_0^2}$

For (11), $H1 = \frac{1}{1+g_0} = H2$

For (9), $T1 = \frac{\theta_1}{v_0}$

In order to obtain the supermultipliers of the above system one has to replace the equations (3), (6),(9),(12),(14), (15), (18) and (19) into the system.