Banking and Leverage: Toward a Model of Financial Stability

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This paper presents a theoretical model of bank leveraging based on Stein (2011, 2012) and Brunnermeier and Sannikov (2014). The paper offers an analytical review of the literature on bank leveraging and focuses on the adverse effects of excessive leverage and the vulnerabilities and credit contractions that can subsequently occur. Moreover, this paper solves the optimal control problem of the model with a finite time decision horizon to present an analysis of the dynamics of net worth and optimal or sustainable leverage, including a discussion of periods of high and low interest rates. The study shows that overleveraging and optimal debt move in opposite directions. Furthermore, a literature review on bank leverage and monetary policy is presented followed by an analysis of the link between credit growth, GDP growth, and overleveraging in five countries. The findings show that periods of low credit growth are accompanied by periods of low GDP growth because constrained credit limits investments as well as consumption in many cases.

Keywords: banking instability, banking sector, real economy, credit flows, financial crisis, debt crisis, excess debt. Non-linear Model Predictive Control

INTRODUCTION

Over the past two decades, an ever-increasing level of bank leveraging has occurred, which has been one of the main causes behind the financial market collapse. This behavior in the banking industry accompanied several trends in the real estate sector, where the subprime mortgage crisis originated. For example, the home price-to-rent ratio began to increase rapidly in 2001 (Finicelli, 2007) and American households accrued debt at an unprecedented level (Hudson, 2006). Interestingly, 90% of this increase in debt since the 1990s consisted of mortgage loans, which actually constituted 50% of total bank loans in general (Semmler, 2011). This increase in mortgage loans was caused by the low interest rate, which increased home buyers' borrowing capabilities. In addition, leveraging increased due to the expanding role of financial intermediaries that acted in the middle and supposedly hedged risk for profit. Because risk was transferred through the securitization of risky loans, banks increased their risk-taking behaviors and neglected screening borrowers for risks.

This essay discusses whether and why excessive leverage leads to instability in the banking sector through adverse shocks in asset prices. The purpose of this paper is to concisely survey the literature on banking leverage and to present a model of optimal leveraging along with its solution, with the application

of a regime-switch. The paper examines and presents supportive results for the hypothesis that leveraging increases when the interest rate decreases and when credit constraints are severe.

Researchers have concluded that the financial market crisis of 2007–2009 resulted from both the typical mechanism of a boom–bust cycle and the financial market culture of excessive leveraging. This paper focuses on the effects of excessive leverage in the banking sector. Modern banking systems have transitioned from traditional banks to complex investment banks that engage in risky trading for profits. As Semmler (2011) argues, deregulation, liberalization of capital accounts in many countries, and the development of new financial instruments and derivatives are the main reasons behind the growth of financial markets. Recently, adverse shocks in asset prices have contributed to the banking crisis because banks leveraged more than a safe amount; in other words, banks used borrowed capital for investments in risky financial markets in pursuit of high profits.

This paper is organized into the following subsections. Section 2 comprises a literature review of leverage in the banking industry. Section 3 introduces the primary theoretical model of bank leveraging. Section 4 utilizes the Non-linear Model Predictive Control (NMPC) to provide a solution for the theoretical model while allowing for a regime change for the bank's interest rate. Section 5 empirically examines the link between excess leverage and several important macroeconomic variables. The summary and conclusion are presented in Section 6.

LITERATURE REVIEW

This paper contributes to recent academic research on the topic of excessive leveraging and the effects of large shocks on asset prices, financial markets, and the balance sheets of banks that might be destabilizing rather than mean-reverting. According to economic theory, lending booms may precede banking system instability because they imply increased risk-taking in a financial system that has the potential to result in financial turmoil if the economy is hit by a negative, adverse shock in asset prices, which is what occurred during the recent crisis. It is important to specify the meaning of banking instability. Jokipii et al. (2013) define instability as the probability of the banking sector becoming insolvent within the next quarter. They state that if at the end of the quarter the market value of the assets owned by all the banks of a certain country is not sufficient to repay its total debt, then the entire banking sector is considered insolvent. Moreover, they define distance-to-default as the distance between the banking sector and its default point (in this case, assets are equal to liabilities). Moreover, the authors argue that there are strong links between banks. Therefore, through distance-to-default, banks may be vulnerable due to the country-specific, timevarying covariance matrix and the entire banking sector of a specific country is then considered insolvent (contagion effect). Many studies have investigated issues related to the asset price channel through which the banking system's instability is triggered. Some of the important academic contributions discussed in this paper include Brunnermeier and Sannikov (2014), Mittnik and Semmler (2012a, 2013), and Stein (2011, 2012).

For instance, the study by Brunnermeier and Sannikov (2014) focuses specifically on the banking sector. The authors state that a shock to asset prices creates a vicious cycle through the balance sheets of the banks. In other words, risk-taking and excessive borrowing occur when asset prices are volatile. They define what they refer to as the volatility paradox as the shock to asset prices that negatively affects banks' balance sheets and, subsequently, disrupts the real

sector. Thus, when the prices of banks' assets decrease, their equity value and net worth decrease and the margin loan requirements increase. For financial intermediaries to remain liquid, they take shortcuts and de-leverage. Consequently, a fire sale of assets begins, decreasing the asset price further and diminishing net worth, thus triggering an endogenous jump in volatility and a risk for all that generates a downward spiral.

According to Mittnik and Semmler (2012a, 2013), the unconstrained growth of capital assets through excessive borrowing facilitated by the lack of regulations imposed on financial intermediaries is considered the main cause for banking sector instability. On the other hand, large payouts with no "skin in the game," affect banks' risk-taking behaviors, equity development, and leveraging. The higher the payout, the more

leveraged the bank becomes, which increases the aggregate risk and risk premia for all. In summary, the increased risk spreads and risk premia, especially at a time when defaults begin, expose banks to vulnerabilities and financial stress triggered by security price movements.

Stein (2011, 2012) argues that the destabilizing mechanism also results from a link between asset prices and borrowing. He specifies that overleveraging begins when assets that are held by banks become overvalued. Above-average returns, due to housing prices that increase the owners' equity, induce a greater demand by banks for mortgages and funds, and, thus, banks enjoy capital gains above the normal returns. At this point, banks begin to become overleveraged compared with optimally leveraged. The basis of Stein's model is that the optimal capital structure reflects the threshold beyond which net worth declines. His analysis is based on the assumption that the mean interest rate exceeds long-term capital gains, a constraint that he refers to as "no free lunch." Therefore, for overleveraging to occur, a violation of this constraint should take place: the capital gain should be larger than the financing cost, providing the banks with excess returns on capital and a high net worth. On the other hand, if capital gains decrease, the credit spread increases, actual leveraging significantly deviates from optimal leveraging, a rapid deterioration of the balance sheets of banks occurs, and amplified downward effects are triggered. Stein suggests using the trends/drifts in capital gains and interest rates to better measure optimal debt.

He also defined excess debt as the difference between the actual and optimal debt. The Stein model is discussed in more detail in the next section.

Researchers have considered the financial market meltdown of 2007–2009 as a result of both the typical mechanism of a boom–bust cycle as well as the financial market culture of excessive risk-taking, leveraging, and risk transfer. In this paper, the focus is on the second cause, with the banking system being the driving force behind assets price swings, crises, and economic instability.

He and Krishnamurthy (2008) study the role of financial intermediaries in determining asset prices and find through their dynamic general equilibrium framework that the need for intermediation arises endogenously. They state that during crises and periods of asset swings, the capacity of risk-bearing of the marginal investors, who are the financial intermediaries in this case, is reduced. The authors replicate the observed rise during crises in Sharpe ratios, conditional volatility, correlation in price movements of assets held by the intermediary sector, and the fall in interest rates.

Many studies have demonstrated that the US financial crisis of 2007–09 was caused by excessive leveraging of the US banking sector and its exposure to the US real estate sector. Schleer, Semmler, and Illner (2014) go beyond that to study the spillover effect of leveraging on the real sector. The authors start from the academic literature assumptions that an overleveraged banking sector leads to constraints in credit supply and recovery slows down. They find that in the few years preceding the financial crisis, actual and optimal debt deviated and the banking sector began to suffer from overleveraging.

Similarly, Mittnik and Semmler (2011) apply DSGE to determine the amplifying effect of the financial sector on real economic activity. Empirically, this is often shown in a one-regime VAR (Gilchrist et al., 2009; Christensen and Dib, 2008; Del Negro et al., 2010); however, the authors employ a multi-regime vector autoregression (MRVAR) approach. They use a non-linear (MRVAR) to explore the consequences of instabilities arising from regime-dependent shocks. They employ data on industrial production and the IMF Financial Stress Index for eight economies, namely, the US, Canada, Japan, and the UK, and for the four largest euro-zone economies, namely, Germany, France, Italy, and Spain.

The authors find a non-linear positive relationship between the real sector and the financial sector stress, but individual risk drivers affect economic activity differently across stress regimes and across countries.

While multi-regime models give a unique approach to the effect of leveraging on the macroeconomy, some one-regime VAR models give significant results and explanations. For instance, Christensen and Dib (2008) use US data post-1979 and estimate two DSGE models, one with and one without a financial accelerator, in order to analyze the effect of the latter on the strengthening of macroeconomic instability. The authors' results suggest that the financial accelerator has a significant role in amplifying the effects of demand shocks on investment and reducing those of supply shocks while not affecting the output fluctuations much. Gilchrist et al. (2009) also estimate a DSGE model with the financial accelerator to study

the link between agents' financial health and the amount of borrowing. They provide empirical evidence that financial frictions have played a crucial role in US cyclical fluctuations.

Moreover, Adrian and Brunnermeier (2009) tackle the issue from another angle. They suggest a new measure for systemic risk that is restricted to institutions or even the whole financial sector not suffering from financial distress. On the other hand, several researchers have studied the contagion effect of financial instability. For example, Boyson, Stahel, and Stulz (2008) find that the average probability that a hedge fund style index suffers from extreme poor performance increases when the number of other hedge funds with extreme poor performance increases.

Furthermore, recent papers have also linked the contagious effects of financial instability to financial innovation. For instance, Jerome L. Stein's explained the dynamics behind the global financial crisis in both the US and Europe. In his two articles (2011, 2012), he states that there are both similarities and differences between the origins of the debt crisis in the US and Europe, depending on which regions within Europe are under consideration. In discussing the origins of the crash in the US and later in Ireland and Spain, Stein focuses his discussion on optimal debt ratios and early warning signs of economic distress in an attempt to explain the role of "excess debt." The reasons for this form of analysis are brought up in his description of the chronological order of events leading up to the crisis and the mechanisms at work in the disaster. For example, he emphasizes the importance of the housing sector in terms of increasing observations of above-average capital gains on the purchasing and subsequent sales of properties in the US in the decade preceding the financial crisis. Above-average returns, due to housing prices and then increased owner equity, induced greater demand by banks for mortgages and funds.

Stein (2012) states that most of these subprime mortgages in 2006 were derived for the purpose of refinancing an existing mortgage loan into a larger mortgage loan, in which payments were anticipated to be made from capital gains. One must understand that the driving force behind the institutional strategy was to increase the supply of such instruments and, in turn, the ratio of household debt service-to-disposable income.

In the same years preceding the economic crisis of 2008, the securitization of mortgages became an increasingly popular and profitable way for financial intermediaries supposedly to hedge risk and increase leverage. Hence, a slew of complex financial derivatives called Mortgage Backed Securities (MBSs), Credit Default Obligations (CDOs), and Credit Default Swaps (CDSs) began to emerge with increased usage and demand, which fueled increasing home values while decreasing the quality of such instruments as the economic downturn approached. These instruments relied on short-term financing from the shadow banking industry, in which investment banks were subject to less financial regulation and relied on internal and overly optimistic risk models to determine capital requirements (Stein 2012; Semmler 2011).

Ratings agencies in the US frequently gave triple-A ratings to these securities and overseas capital flooded the market in search of safe investments that delivered higher returns than government debt. This flow of funds fueled the financial system to design debt instruments of varying risk quality and selling multiple tranches of such security issues. Essentially, Stein describes the US housing bubble as being financed by an increased flow of capital. Once firms began to use these instruments to increase leverage for profitable endeavors, problems could be found in the fact that these institutions had the latitude to decide that their capital requirements for certain tranches of CDOs being held for trading purposes were essentially zero. As long as home prices kept increasing, the effect on the system's balance sheets was positive, which helped drive the boom in the housing market for years. However, the interconnectedness of the financial system presented potential problems when the above-average increases in housing prices could no longer be maintained.

By 2007, many households were unable to service their debt requirements and, as defaults began to emerge in greater quantities, housing prices began to deteriorate across the US. Before elaborating on the mechanism through which changes in the balance sheets of institutions cause deterioration in market prices resulting in a financial crisis, it is important to note that Stein does not attribute the crisis to high debt, but he does define excess debt in relation to his

Similarly, Adrian and Shin (2009) studies the impact of securitization on financial stability. The authors find that securitization, which increases leverage, loosens lending standards and exposes the financial sector

to vulnerabilities. Moreover, Reinhart and Rogoff (2009) have contributed several papers analyzing the effects of banking crises. They employ data from high and middle-to-low income countries and show that credit booms and asset price bubbles usually precede a systematic banking crisis. Furthermore, Herring and Wachter (2003) demonstrate that several financial crises are the result of bubbles in real estate markets; more specifically, crises result, on average, "in a 35% real drop in housing prices spread over a period of 6 years."

Before concluding this section, it is important to note that the body of literature regarding the events preceding and during the current financial crisis is very large, and this paper includes only a few with the focus on the opinions and models of Jerome Stein. However, in future research, it is worth examining additional papers such as Adrian and Shin (2009), Brunnermeier (2009), and Greenlaw et al. (2008). These papers attribute the origination of the financial crisis to the low interest rate policies adopted by the Federal Reserve and other central banks, which was a result of the technology stock bubble failure.

A MODEL OF BANK LEVERAGING

In discussing the origins of the crash, Stein (2012) focuses his discussion on optimal debt ratios and early warning signs of economic distress in an attempt to explain the role of "excess debt."

Net Worth Dynamics

According to Stein (2012), excess debt is defined as follows:

$$\psi(t) = EXCESS\ DEBT = N[\mathscr{F}(t)] - N[\mathscr{F} ** (t)]$$

where N $[\mathscr{F}**(t)]$ = An upper bound on the optimal debt ratio Hence, Stein defines the terms as follows:

$$\psi(t) = Normalized \left\{ DEBTSERVICE \frac{i(t)L(t)}{Y(t)} - RENTPRICER \frac{R(t)}{P(t)} \right\}$$

Essentially, Stein is utilizing methods of analysis used in deriving the optimal debt ratio by describing the probability of a decline in net worth as being positively related to excess debt. His analysis is based on the assumption that the mean interest rate exceeds the long run capital gains, an assumption that was violated in the decade preceding the financial crisis.

To elaborate on Stein's model and the mechanisms at work, it is helpful to understand his description and determination of optimal debt ratios in the models that he utilizes in both papers, which are closely related. To begin with, Stein uses an approach called "stochastic optimal control." In such a problem, the goal is to find an optimal debt ratio, $f^*(t)$, that maximizes an investor's net worth at a terminal time horizon. However, this solution is subject to stochastic processes with regard to the housing price and interest rate, and the control variable is the optimal debt ratio.

Stein defines net worth, X(t), as assets minus liabilities and then defines his primary equation for the Change in Net Worth as a stochastic differential equation as follows:

$$dX(t) = X(t) \left[\left(1 + f(t) \right) \left(\frac{dP(t)}{P(t)} + \beta(t) dt \right) - i(t) f(t) - c dt \right]$$
(1)

In this case, he defines the Debt Ratio as:

$$F(t) = \frac{L(t)}{X(t)} \tag{2}$$

Capital Gains or Losses are:

$$\frac{dP(t)}{P(t)} \tag{3}$$

Productivity of Capital is:

$$\beta(t) = \frac{Income}{Assets} \tag{4}$$

Ratio of Consumption is:

$$\frac{(Consumption or Divedends)}{New Worth} \tag{5}$$

where i(t) is the interest rate. Stein then sets up an equation for price, which is composed of a trend and deviation from that trend (r) as follows:

$$P(t) = Pexp(rt + y(t)) \tag{6}$$

The deviation from the trend is demonstrated through:

$$y(t) = \ln P(t) - \ln P - rt \tag{7}$$

The mean reversion aspect characterized by a convergence of α , is defined as:

$$dy(t) = -\alpha(t)dt + e_p dw_p(t) \tag{8}$$

In this model, Stein defines E(dw) = 0; $E(dw)^2 = dt$

$$limy(t) \sim N\left(0, \frac{\sigma^2}{2a}\right) \tag{9}$$

Stein constrains the solution such that $r \le i$ and calls this the "No free lunch constraint."

Hence:

$$dP(t) = P(t) \left(a(t)dt + e_p dw_p(t) \right) \tag{10}$$

where a(t) represents the asset's drift component and the interest rate is represented by the sum of i and a Brownian Motion term as follows:

$$i(t) = idt + e_i dw_i(t) \tag{11}$$

Substituting through the solution, Stein arrives at and defines the optimal debt ratio as follows:

$$\mathcal{F}^*(t) = \left[(r-i) + \beta - ay(t) - \frac{\left(\frac{1}{2}\right)(\sigma_p^2 - \sigma_i \sigma_p \rho)}{\sigma^2} \right]$$
 (12)

where ρ represents the negative correlation coefficient. The full mathematics of the Stein model (2012) are found in Appendix A.

Through the model presented above, Stein is able to determine the excess debt and an early warning signal of a potential crisis. It is this mechanism that played a part in the decreasing net worth of individuals,

households, and institutions in the US and was amplified by the increased leverage and pricing volatility of complex securities.

Therefore, in the US, as delinquencies and defaults resulted in ratings agencies downgrading various complex securities in 2007, prices deteriorated and resulted in fire sales of assets as institutions scrambled to unload the now risky assets. Mark-to-market pricing resulted in greater uncertainty about the future and herd mentality drove the cycle to continue. In the process, some of the largest US financial institutions, such as Bear Stearns and Lehman Brothers, faced collapse as increasing liabilities relative to assets put the firms in danger of Chapter 11 bankruptcy. The damage to the short-term repo market created substantial systematic risk to the financial system, which in 2007–2008 had a contagion effect on European nations that were, for various reasons, in delicate financial positions in their own right.

Mean-variance Principle to Compute Sustainable (Optimal) Debt

The Stein model can also be viewed as a special application of the optimal portfolio choice problem. As mentioned before, during the financial crisis, it was observed that as asset prices increased, they were used for increased leveraging, which could increase expected returns as well as volatility from an economic shock. Therefore, the optimal debt problem can be viewed in terms of maximizing a linear combination of capital gains and the level of risk similar to a portfolio decision.

In terms of a maximization portfolio decision we have:

$$\max_{a_t} \left[a_t \left(E(R_{t+1}) - R_{F,t+1} \right) - \frac{k}{2} \alpha_t^2 \sigma_t^2 \right] \tag{13}$$

In the above equation, the first term represents the market risk premium and the secondterm represents the risk or variance of the investment. In other words, I may define the Stein model with an M term, representing risk adjusted returns and an R term, representing risk. Therefore, I may write:

$$f^* = argmax_f [M(f(t)) - R(f(t))] = f^*(t) = \left\{ (r - i) + \beta - ay(t) - \frac{1/2(\sigma_p^2 - \rho_{ip}\sigma_i\sigma_p)}{\sigma^2} \right\}$$
 s.t. (14)

$$Risk = \sigma^2 = \sigma_i + \sigma_p - (2\rho_{ip}\sigma_i\sigma_p)$$
(15)

where P is the portfolio and ρ is the negative correlation coefficient.

Alternatively, his second model may be similarly derived via a Brownian Motion component for capital gains where:³

$$\frac{dP(t)}{P(t)} + \beta(t) = \pi dt + \beta(t) + \sigma_p dw_p \tag{16}$$

Once again, the negative correlation is represented by ρ ; thus, as the interest rate falls the capital gains rise (and vice versa). This may be written similarly with regard to returns and risk as follows:

$$f^* = argmax_f \left[M(f(t)) - R(f(t)) \right] = F^*(t) = \left\{ (r - i) + \beta - ay(t) - \left(\frac{1/2(\sigma_p^2 - \rho_{ip}\sigma_i\sigma_p)}{\sigma^2} \right) \right\}$$
(17)

s.t.

$$Risk = \sigma^2 = \sigma_i + \sigma_p - (2\rho_{ip}\sigma_i\sigma_p)$$
(18)

Clearly, $f^*(t)$ is negatively related to risk terms and deviations from capital gains as in a mean-variance portfolio decision problem. This form of analysis can be used to determine the excess debt using time series

data, which can be an early warning signal of economic distress as Stein describes when discussing the real estate and banking crisis of 2008.

NMPC Solution of the Model

In this paper, NMPC is used to solve the optimal control problem with a finite time decision horizon. NMPC relies on iterations and only computes one optimal trajectory at a time and a global solution through full non-linearization (Grune, Semmler, and Stieler, 2015) and it can track the dynamics of the state variables.⁴

This section focuses on the behaviors of financial institutions, particularly in relation to the size and timing of interest rate jumps. The framework considers the instability of financial institutions and the relevant macroeconomic downturns. The NMPC solutions presented are applicable to extremely long horizons, approaching infinite-horizon solutions. This model allows for empirical measures on and estimation of overleveraging, which are applied in the next chapter using time series data at the banking level

The starting point for this model of banking leverage and net worth dynamics is a low-dimensional stochastic model specification. As mentioned, the model discussed in this paper is similar to the models used by Stein (2012, 2013), Brunnermeier and Sannikov (2014), Shleer and Semmler (2015), and Gross et al. (2016).⁵ The Stein model comprises two decision variables, namely, leverage f t and payouts ct, and one main state variable, which is net worth, $c_{1,t}$. Here, the focus is the solution of the dynamic version with shocks that displays the mechanism of overleveraging. To solve a stochastic variant model through NMPC, a stochastic shock sequence, $x_{2,t}$, must be added; hence, Equation (21) was added, representing another state variable. In Stein (2012), both the capital return and the interest rate are stochastic as well, whereas in Brunnermeier and Sannikov (2014), only the capital return is stochastic.

As mentioned, the framework is modeled as a finite horizon⁶ decision problem with a decision horizon of N in discrete time⁷. The model can be presented as follows:

$$V = \max_{c_t f_t} E_t \sum_{t=0}^{N} \beta^t U(c_t, x_{1,t})$$
 (19)

s.t.

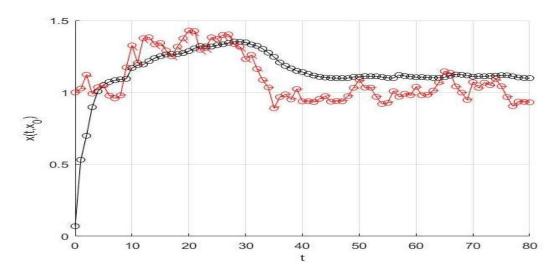
$$x_{1,t+1} = x_{1,t} + hx_{1,t} [(1+f_t)(y+v_1 ln x_{2,t} + r) - (i-v_2 ln x_{2,t} + r)f_t - a\varphi(x_{1,t}) - c_t]$$
(20)

$$x_{2,t+1} = \exp(\rho \ln x_{2,t} + z_k) \tag{21}$$

where V is the value function, U(.) is an instantaneous utility function and depends on the flow of consumption C(.), and c and f are the two decision variables, with $c = \frac{c}{x_1}$ and $f = \frac{d}{x_1}$ (denotes debt). Moreover, p denotes capital gains, driven by stochastic shocks, $p_1 \ln x_{2,t}$, and $p_2 \ln x_{2,t}$, and $p_3 \ln x_{2,t}$ is the return on capital. In the solution of the dynamic Equation (20), $p_3 \ln x_{2,t} = \frac{d}{d} \ln x_{2,t} = \frac{d$

Figure 1 exhibits the paths of net worth $x_{1,t}$ modeled by Equation (20) and the stochastic process modeled by Equation (21), with initial net worth condition below the stochastic steady state.

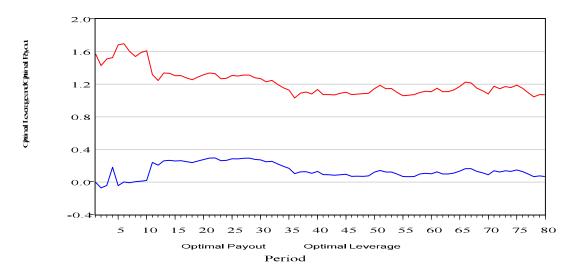
FIGURE 1
PATHS OF NET WORTH $x_{1,t}$ MODELED BY EQ. (20) (CHAIN OF CIRCLES) AND STOCHASTIC PROCESS MODELED BY EQ. (21) (LINE WITH CIRCLES) WIT THE INITIAL NET WORTH CONDITION BELOW THE STOCHASTIC STEADY STATE



As shown in figure 1, when the Period chastic shock, which causes the volatility of leverage, increases, net worth increases; therefore, leverage and net worth move in the same direction, and payouts fluctuate roughly with leverage. It should be specified that leverage is optimal, not actual, as modeled by Stein (2012). As mentioned by Shleer et al. (2016), actual leveraging can move in a different direction than optimal leveraging because it is driven by other factors; however, optimal leveraging slightly depends on actual leveraging. Stein (2012) emphasizes the difference between actual, optimal, and excess leverage without extending his analysis to the macroeconomic effects of overleveraging, which are covered in this section.

Figure 2 depicts the paths of optimal payout, c_t , and optimal leveraging, $\alpha_t = (1 + f_t)$

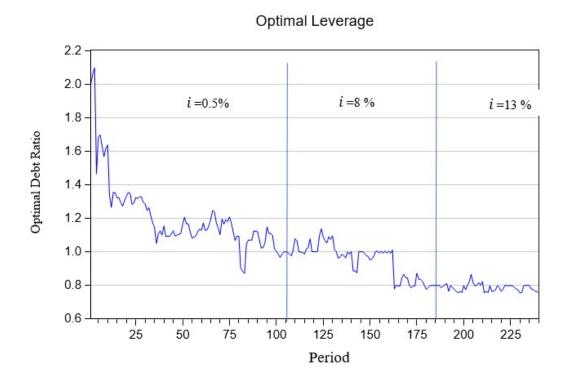
FIGURE 2
PATH OF OPTIMAL PAYOUT, ct, (LOWER LINE) AND OPTIMAL LEVERAGING, $\alpha t = (1 + ft)$ (UPPER LINE)



In the graph illustrated in Figure 2, leveraging, $\alpha_t = (1 + f_t)$, is always above one using simulated f_t data generated in MATLAB, implying no excess leverage conducted by banks. Similar to Brunnermeier and Sannikov (2014) and Stein (2012), debt is assumed to be redeemed in each period, and without cost it can be obtained in the market without friction.⁸

Figure 2 reinforces the finding from the figure before. Clearly, leverage and payout move together in the same direction. Next, regime change for the main banks' variable, namely, the interest rate paid on bank liabilities, is allowed for. Graph 3 shows the effect of the interest rate change on optimal leverage. Theoretically speaking, when the interest rate is low, the negative expression in Equation (20) $(i - v_2 ln x_{2,t})$ becomes smaller, increasing the first positive term and, in turn, increasing α_t and $x_{1,t+1}$. The opposite occurs when the interest rate is high. In conclusion, the interest rate has a large impact on leveraging in both cases, whether it is a low interest or a high interest rate, as shown in Figure 3, which depicts the optimal leverage for three periods of regime changes in the interest rate.

FIGURE 3
REGIME CHANGES IN THE INTEREST RATE AND SHIFT IN OPTIMAL LEVERAGING, $\alpha t = (1 + ft)$: LINE UP TO PERIOD 80, LEVERAGING FOR INTEREST RATE=0.5%; LINE FROM PERIOD 81 UP TO PERIOD 160, LEVERAGING FOR INTEREST RATE=8%; LINE FROM PERIOD 161 UP TO PERIOD 230, LEVERAGING FOR INTEREST RATE=13%



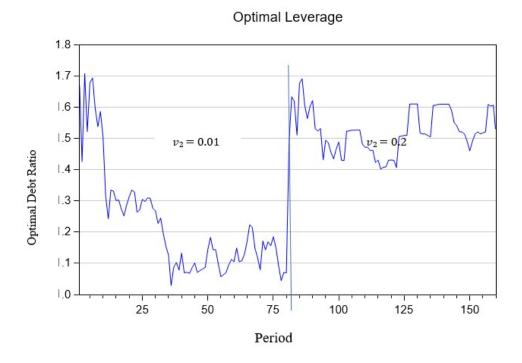
As the graph illustrates, the interest rate and optimal leveraging move in opposite directions. During the first period, the interest rate was equal to 0.5% and the optimal leverage was above one throughout the period. On the other hand, the optimal leverage dropped to one or slightly below one for the 8% interest rate and dramatically decreased to an average of 0.8 when the interest rate jumped to 13%.9 Because the components of the optimal debt ratio are primarily the capital gains for equity holders of the banks' stocks, the market interest rate, and the risk term, the optimal debt ratio increases the excess debt during high interest rate periods for a given level of actual leveraging. Moreover, as mentioned, the optimal debt ratio maximizes the difference between the mean return and the risk term; thus, an interest rate increase exposes

banks to additional vulnerabilities due to the excess leverage caused by the decline in sustainable debt. Basically, when the interest rate is high, banks must generate profits to pay their debts. Banks then leverage in pursuit of profit and by doing so expose themselves to higher risks.

The model shown in Figure 4 is solved using NMPC in a stylized manner, where the parameter v2 is changed to examine the effects of the size and timing of interest rate jumps through changes in the shock sequence.

The shock sequence $X_{2,t+1} = \exp(\rho l n_{X2,t} + Z_k)$ modeled by Equation (21) is equal to $X_{2,t} + \rho + e^{zk}$. As shown, it depends on $x_{2,t}$, which in turn is multiplied by the parameter v_2 in Equation (20). Thus, it would be interesting to change the parameter v_2 and examine the effect of the shock sequence on leveraging. Theoretically speaking, when v_2 increases, this negative expression in Equation (20) $(i - v_2 l n x_{2,t})$ becomes smaller, increasing the first positive term and, in turn, increasing (1 + ft) and $x_1,t+1$. The opposite occurs when the shock is less persistent. In other words, an effect opposite to the effect of the interest rate change is expected. This can also be intuitive because the interest rate and the parameter v_2 have opposite signs. ¹⁰

FIGURE 4
CHANGE IN THE SHOCK SEQUENCE AND SHIFT IN OPTIMAL LEVERAGING $\alpha_t = (1+ft)$ LINE UP TO PERIOD 80, LEVERAGING FOR v2=0.01; LINE FROM PERIOD 81 UP TO
PERIOD 160, LEVERAGING FOR v2=0.2; INTEREST
RATE WAS SET AT 2% FOR BOTH CASES



Graph 4 illustrates that a jump in the parameter v_2 led to a jump in optimal leveraging and eventually in net worth. When there is a shock, the interest rate remains steady or decreases, which increases the optimal leverage.

As discussed, the interest rate has a considerable impact on leveraging in both cases, whether it is a low interest or high interest rate. Generally speaking, bank stakeholders' main interest is the return on equity because it is the return they earn on their investments in which assets play a major role. The return on assets as well as the total value of the assets generate income for the banks, and the banks' profits are not limited by the fees they can charge. This has caused banks to purchase more assets and to primarily pay for them

with more liabilities and with less bank capital. Paying with banks' capital negatively affects the banks' balance sheets, which is why modern banks engage in high leverage.

With this leveraging dynamic, banks' liabilities increase to cover the assets' costs, but the interest rate spread is limited by what a bank must pay on its liabilities and what it can charge on its assets. In a world of competition and a race for profits and survival, banks must pay a minimum market rate to attract depositors. Likewise, banks can only charge a certain interest rate for loans to attract borrowers. Hence, interest rate spreads are not wide, so a bank's profit can only increase by selling more loans and earning more net interest. With high leverage comes high risk, and the vicious cycle begins again. (Schleer, Semmler, and Illner. (2014)) question whether banks should expect such a cycle and avoid being overleveraged to minimize risk; however, they attribute this to the shortsightedness of the banks, which do not assess risk vulnerabilities accurately.

Therefore, regulations play an important role in protecting the financial sector and the banking system. For instance, the Federal Reserve restricts the amount of leverage that banks that are depository institutions can use. Typically, the leverage ratio is about 10 to 12 times; in other words, a bank's assets may have at least 10 times the value of its capital but cannot exceed this amount.¹¹

LEVERAGE AND MACROECONOMIC INSTABILITY

Background

There is a large body of literature on the link between interest rates and banks' credit standards, which has led to an increase in the supply of complex financial derivatives, namely, Mortgage Backed Securities (MBSs), Credit Default Obligations (CDOs), and Credit Default Swaps (CDSs). ¹² In this section, the literature on the link between leveraging and monetary policy is concisely surveyed, and then the link between credit growth, GDP growth, and overleveraging in the banking sector in five economies is analyzed.

To introduce the topic of bank leverage and monetary policy, several contributions to the field are discussed. For instance, Maddaloni and Peydro (2011) examine the effect of short-term interest rates on lending standards in the US and Europe. Using data on European and American lending standards from bank lending surveys, they find statistically significant evidence that credit standards are loosened when overnight rates are lowered. Moreover, using Taylor rule residuals, they find that holding rates low for prolonged periods of time softens lending standards even further. Allen et al. (2009) and Rajan (2009) suggest that the same relationship exists between real interest rates and risk-taking by banks.

Furthermore, Dell'Ariccia et al. (2013) depart from the previous framework and use confidential data on the internal ratings of US banks on loans to businesses from the Federal Reserve's survey of terms of business lending. The empirical strategy relied on a series of regressions spanning the period from 1997 to 2011. The authors find a negative relationship between monetary policy tightening and bank risk-taking.

Admati (2013) similarly contend that credit is highly affected when a certain threshold of leverage ratio is reached, but they added output to their analysis. They state that credit flows are constrained when the banking sector suffers from overleveraging, which has a contagious effect on the real sector. The authors raised broad critical questions regarding bank regulations and argued that it is possible to have a safer banking system without sacrificing lending and economic growth. Their main message can be summarized as the need for banks to have more equity or capital to be more stable. The authors did not extend their analysis beyond the hypothesis and did not include an empirical analysis.

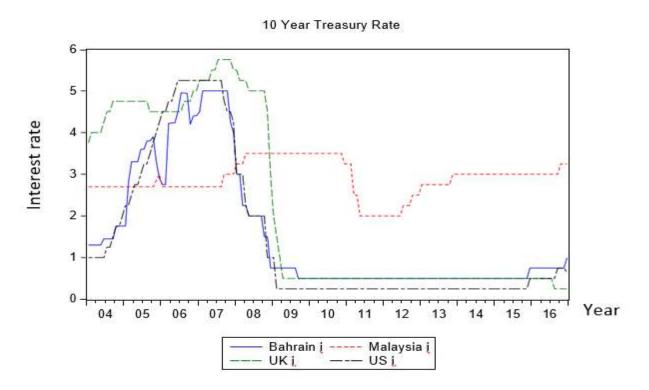
On the other hand, Shleer, Semmler, and Illner (2014) empirically test the hypothesis that the overleveraging of banks negatively affects the financial and real sectors and exerts pressure on credit flows to the private sector. The authors use a non-linear Vector STAR model to examine the link between credit and GDP in economies based on overleveraging and an exogenous regime-determining variable, which they calculate based on Stein's (2012) optimal debt equation mentioned. Thus, using the calculated time series data on overleveraging for 40 European banks from 1997 to 2012, Shleer et al. (2014) examine the effects on credit and output during periods of overleveraging as well as periods of optimal or sustainable debt. Their findings reveal that there is a strong positive correlation between credit and GDP growth.

Moreover, Dell'Ariccia, Detragiache, and Rajan (2008) provide evidence that bank vulnerability leads to a decline in credit growth and to low GDP growth. They conduct a comparison between sectors that are more dependent on external finance and those that are less dependent and find that the dependent sectors performed relatively worse during banking crises, especially in developing countries, which have less access to foreign finance. Similarly, Detken et al. (2014) find that there is low credit growth during periods of low GDP growth, but did not extend their analysis to investigate the effects of or the link with excessive leveraging.

Stylized Facts and Correlation Analysis

In this study, the hypothesis that excess leveraging negatively affects GDP and credit growth was empirically tested, as was done by Shleer et al. (2016). Excess leveraging has been calculated for 20 banks in Bahrain, Kuwait, Malaysia, the US, and the UK13 and, in this section, it is used as the measure for overleveraging. Quarterly data of GDP and credit growth were used and the annual measure of excess debt (leverage) was converted through the quadratic match average method so that all time series data were available at a quarterly frequency. The method fits a quadratic polynomial used to fill in all observations of the high frequency series, such that the averages of the annual and quarterly series match. Before presenting the results of the correlation analysis, a plot of the interest rate behaviors for four countries is presented.

FIGURE 5
PATHS OF INTEREST RATES IN BAHRAIN, MALAYSIA, THE US, AND THE UK



The graph in Figure 5 depicts the interest rates for Bahrain, Malaysia, the US, and the UK. It shows that Bahrain and the US track each other closely. This is expected because the US leads the market and Bahrain has pegged its currency to the dollar. Moreover, Bahrain's central bank officials have frequently declared to the media that they raise the key interest rate based on the US Federal Reserve policy. Many countries in the Gulf Cooperation Council (GCC) area follow the US when setting rates. As shown, rates began to increase in 2014, reaching a high of 6% around the years 2006 and 2007, then sharply decreasing

beginning at the end of 2008. The UK interest rate also fluctuated around the same pattern. In contrast, Malaysia's rate was totally independent and had its own pattern. The rate was high between 2007 and 2011, in contrast to most countries in the world, and decreased in 2012 only to increase again at the end of 2013, and it has remained stable since. In Malaysia, this could be due to lower integration of Malaysia in the global economy relative to the UK.

Table 1 presents the correlation between credit, GDP growth, and the measure of overleveraging in the banking sector for each of the five countries, using quarterly data. The excess leverage measure was taken as the average of a sample of four banks in each country and the GDP growth and credit growth¹⁷ data were extracted from tradingeconomics.com.¹⁸

TABLE 1
CORRELATION BETWEEN CREDIT, GDP GROWTH, AND THE MEASURE OF EXCESS LEVERAGING

	Bahrain	Kuwait	Malaysia	US	UK
GDP & Credit	0.066940	0.273493	0.6880619	0.32	0.457
GDP & Excess Leverage	-0.151977	0.0523742	-0.1487106	0.034	0.248
Credit & Excess Leverage	0.4765438	-0.1691910	0.25416796	0.2333	-0.442

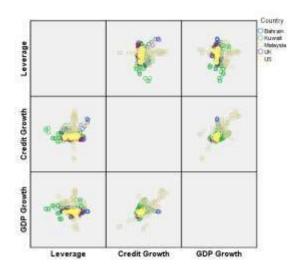
As shown in Table 1, there was a positive correlation between GDP and credit growth for the five countries. This was expected and is in line with all literature studies. It has been proven that periods of low credit growth are accompanied by periods of low GDP growth because constrained credit limits investments as well as consumption. The correlation was highest for Malaysia followed by the UK while the correlation was lowest for Bahrain. This could be because Bahrain is one of the countries where government intervention is strong and enforces credit standards. Moreover, Bahrain is considered a competitive market, especially when compared to most of the markets in the Middle East and North Africa (MENA) region. For instance, a paper by Turk-Ariss (2009) examined the competition structures of the MENA region countries' banking sectors for the time span 2000–2006 and found that Bahrain and Turkey are the only two countries with perfect competitive markets, whereas most of the countries in the MENA region engage in monopolistic competition.

The expected negative correlations between GDP growth and excess leverage and between credit growth and leverage were less pronounced for some of the countries. While there was a negative correlation between GDP growth and excessive leverage for Bahrain and Malaysia, the data failed to show this for the remaining three countries under study. On the other hand, there is only a significant negative relationship between credit growth and leverage for Kuwait and the UK, which indicates that the higher the excess debt in their banking sectors, the lower their credit growth.

A Pearson's correlation study was also performed on the five countries collectively. ²⁰ According to the Pearson's correlation, there is a strong positive and significant linear relationship between credit growth and GDP growth (correlation = 0.633, p-value < 0.001). The conclusions of Kendall's tau and Spearman's rho are in complete agreement. There is a weak but significant negative non-linear relationship between excess leverage and GDP growth (tau = -0.102 with p-value = 0.005; rho = -0.145 with p-value = 0.008). There is a moderate positive non-linear relationship between credit growth and GDP growth (tau = 0.359 with p-value < 0.001; rho = 0.513 with p-value < 0.001). The weakness of the relationships could be because some of the individual countries did not hold the expected correlation. Moreover, there is an issue with the data because the sample of banks is too small to provide an accurate measure of leverage for these countries.

Figure 6 depicts average GDP and credit growth from 2000 until 2016 for the five countries under study.

FIGURE 6 AVERAGE QUARTERLY GDP GROWTH, CREDIT GROWTH, AND EXCESSIVE LEVERAGE, 2000–2016



As shown in Figure 6, GDP and credit growth were highly positively correlated for all countries, and the excessive leverage was negatively correlated with GDP and credit growth for most countries. Appendix B also contains the graphs for the average quarterly GDP and credit growth for the five countries. Bahrain and the US seem to have a similar pattern where there is high credit compared to GDP growth, especially during the financial crisis between 2007 and 2009; however, Bahrain had an astronomic increase in credit growth between 2007 and 2009, which was much stronger than that in the US. Kuwait had a different outcome. Generally speaking, credit and GDP growth fluctuated around each other for most of the period except during the financial crisis when credit increased dramatically compared to GDP, which did not begin to increase until 2010. For Malaysia, credit and GDP growth had the same pattern, with credit growth being slightly higher. After 2010, the spread between credit and GDP grew wider. The UK showed the best results. According to the graph, there is a strong relationship between credit and GDP growth. This is true because the UK suffered less during the financial crisis than the other countries in the sample. As per Shleer et al. (2015), the data show that high credit growth was accompanied by low GDP growth in the countries that suffered most from the financial crisis.

CONCLUSION

This paper described the issue of excess leveraging and its adverse effects, which most researchers consider to be the main cause for the financial crisis of 2007–2009. As discussed, overleveraging exposes banks not only to high profit but also to high risk. This was first demonstrated through the theoretical model of bank leverage developed by Stein (2012). For Stein, excess debt, rather than actual debt, can serve as an early indicator of crises because the presence of excess debt is actually the reason why banks collapse. Next, the model was solved using NMPC with a special focus on the behaviors of the banks, particularly in relation to the size and the timing of interest rate jumps. The impact of the interest rate changes on leveraging is examined in two ways. First, the interest rate increased, and second, the shock sequence affecting the interest rate increased. In both cases, there was a decrease in optimal leveraging accompanying an increase in the interest rate, whereas excess leveraging increased when interest rates were low. When the interest rate is low, borrowing dramatically increases, stock prices increase and, therefore, leveraging increases, which is evident based on the dynamics of the recent financial crisis. On the other hand, when

the interest rate is high, the credit that is servicing the debt would be expensive. In short, credit booms can cause fluctuations in interest rates and can cause countries to become vulnerable to credit traps.

Finally, Section 5 briefly described the effect of excess leverage on the macroeconomy by discussing the plots of GDP and credit growth for each country and providing the correlation between leverage, GDP growth, and credit growth. The results showed that there is a positive relationship between GDP and credit growth was found for most of the countries in this context. It would be interesting for future research to further explore this relationship and to study the link in a more structural way based on a non-linear Vector STAR model that allows for comparing the economy at periods of low and high leverage. Moreover, although banking vulnerability mainly depends on leveraging, it also depends on a sudden increase in credit spreads and financial stress. Future research can include the financial—macroeconomic link as well as the correlation between the real estate and the financial sectors. Finally, it would be interesting to expand the macroeconomic instability section by examining the effect of both short-term and long-term interest rates on lending standards in emerging markets as per Maddaloni and Peydro (2011).

ENDNOTES

- 1. This source of instability is also discussed by Brunnermeier and Sannikov (201).
- 2. This is what took place a decade before the recent financial crisis
- 3. See Appendix A Model II
- 4. The basic theory of NMPC without discounting was developed in Grune and Pannek (2011).
- All these models are similar in the sense that leveraging is a choice variable and the state variable is net worth. In Mittnik and Semmler (2013a), leveraging is a state variable as well as capital.
- 6. Similar to Stein's (2012) model and contrary to the Brunnermeier and Sannikov (2014) model, which is an infinite-horizon model.
- 7. Brunnermeier and Sannikov (2014) and Stein (2012) employ a continuous time version, but I formulate the problem as a discrete time variant similar to Mittnik and Semmler (2013) and Shleer et al. (2016).
- 8. Because α_t is a decision variable.
- The interest rates were chosen as the average of the highest for the five countries in the sample for 20 years, and the same was done for the average of the lowest rates.
- 10. See Equation (20): $(i v_2 ln x_{2,t})$
- See: https://www.federalreserve.gov/supervisionreg/reglisting.htm
- Mortgages have been grouped together, sliced, and packaged into risky securities of different types, such as CDOs, MBS, and CDS, and the interest income is used to compensate investors for taking positions in which varying levels of default, called "tranches," are guaranteed.
- ^{13.} See Issa (2020): Life After Debt; The effects of overleveraging on conventional and Islamic banks.
- Data and related variable descriptions may be found at the following URLs: https://www.ceicdata.com/en; https://data.worldbank.org/indicator
- 15. "Fit a local quadratic polynomial for each observation of the low frequency series, then use this polynomial to fill in all observations of the high frequency series associated with the period. The quadratic polynomial is formed by taking sets of three adjacent points from the source series and fitting a quadratic so that either the average or the sum of the high frequency points matches the low frequency data actually observed. For most points, one point before and one point after the period currently being interpolated are used to produce the three points. For end points, the two periods are both taken from the one side where data are available. This method is a purely local method. The resulting interpolation curves are not constrained to be continuous at the boundaries between adjacent periods. Because of this, the method is better suited to situations in which relatively few data points are being interpolated and the source data are fairly smooth." See also EViews 9 User Guide provided at the following URL:
 - http://www.eviews.com/help/helpintro.html#page/content/Basedata-Frequency Conversion.html
- For more on this topic, see https://www.reuters.com/article/bahrain-rates/bahrain-central-bank-raises-key-rate-by-25-bps-idUSD5N1KU01V
- 17. Credit growth is the Loans to Private Sector increase.
- 18. https://tradingeconomics.com/bahrain/loans-to-private-sector
- 19. See www.adhrb.org.

- ^{20.} See Appendix A for the results
- ^{21.} For instance, see Mittnik and Semmler (2013).

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APPENDIX A STEIN MATHEMATICAL APPENDIX

Mathematical Derivation of Stein's Optimal Debt

Here, Stein shows how the optimal debt ratio is derived in the logarithm case. The stochastic differential equation is (1):

$$dX(t) = X(t) \left[(1 + f(t)) \left(\frac{dP(t)}{P(t)} + \beta(t)dt \right) - i(t)f(t) - cdt \right]$$
(1)

where the Debt Ratio is $f(t) = \frac{L(t)}{X(t)} = \frac{Debt}{Net\ Worth}$; Capital Gain or Loss is $\frac{dP(t)}{P(t)}$; Productivity of Capital is $\beta(t) = \frac{Income}{Assets}$; Interest Rates is i(t); $i(t) = \frac{Assets}{Net\ Worth}$; Ratio of Consumption is: $c = \frac{(Consumption\ or\ Divedends)}{Net\ Worth}$ and is taken as given.

Let the Price Evolve as:

$$dP(t) = P(t) (\alpha(t)dt + epdw_p(t))$$
(2)

where $\alpha(t)$ represents the asset's drift component and the interest rate is represented by the sum of i and a Brownian Motion term as follows:

$$i(t) = idt + o_i dw_i(t) \tag{1}$$

Substitute (2) and (3) in (1) and derive (4)

$$dX(t) = X(t) \left[(1 + f(t)) \left((\alpha(t)dt + e_p dw_p(t)) + \beta(t)dt \right) - i(t)f(t)dt - cdt \right]$$

$$dX(t) = X(t)(Mf(t))dt + X(t)\beta f(t)$$

$$Mf(t) = \left[(1 + f(t))((\alpha(t)dt) + \beta(t)dt) - i(t)f(t)dt - cdt \right]$$

$$\beta(t) = \left[(1 + f(t))e_p dw_p - e_i f(t)dw_i(t) \right]$$

$$\beta^2 f(t) = \left[(1 + f(t)^2)e_p^2 dt + f(t)^2 e_i^2 dt - 2f(t)(1 + f(t))e_i e_p dw_p dw_i \right]$$

$$Risk = Rf(t) = \left(\frac{1}{2} \right) \left[(1 + f(t)^2)\sigma^2_p dt + f(t)^2 \sigma^2_i dt - 2f(t)(1 + f(t))\sigma_i \sigma_p \right]$$

$$(2)$$

Mf(t) contains the deterministic terms and $\beta(t)$ contains the stochastic terms. To solve for X(t) consider the change in $\ln X(t)$ in (5). This is based upon Ito equation of the stochastic calculus. A great virtue of using logarithm criterion is that one does not need to use dynamic programming. The expectation of dlnX(t) is (6).

$$dlnX(t) = \left(\frac{1}{X(t)}\right)dXt - \left(\frac{1}{2}X(t)^2\right)(dw(t)^2)$$
(3)

$$E[d(\ln X(t))] = [Mf(t)] - R[f(t)]dt \tag{4}$$

The correlation $\rho dt = E(dw_p dw_i)$ is negative, which increases risk. $(dt)^2 = 0$, dwdt = 0. The optimal debt ratio f* maximises the difference between the Mean and Risk.

$$f *= argmax_f[M(f(t)) - R(f(t))] = [\alpha(t) + \beta(t) - i] - \left[\left(\frac{\sigma_p^2 - p\sigma_i\sigma_p}{\sigma^2}\right)\right]$$

$$f^* = argmax_f[M(f(t)) - R(f(t))] = f^*(t)$$
(5)

$$= \left\{ (r-1) + \beta - \alpha y(t) - \left(\frac{1/2(\sigma_p^2 - p\sigma_i \sigma_p)}{\sigma^2} \right) \right\}$$
s.t.
$$Risk = \sigma^2 = \sigma_i^2 + \sigma_p^2 - (2\rho_i p\sigma_i \sigma_p)$$

Model I

Model I assumes that the price P(t) has a trend rt and a deviation Y(t) from it (8). The deviation Y(t) follows an Ornstein-Uhlenbeck ergodic mean reverting process (9). Coefficient α is positive and finite. The interest rate is the same as in model II.

$$P(t) = Pexp(rt + y(t))$$
(8)

The deviation from the trend is demonstrated through:

$$y(t) = ln P(t) - ln P - rt$$

The mean reversion aspect characterized by a convergence of α , is defined as:

$$P(t) = Pexp(rt + y(t)) \tag{9}$$

In this model, Stein defines E(dw) = 0; $E(dw)^2 = dt$

$$\lim y(t) \sim N\left(0, \frac{\sigma^2}{2\alpha}\right)$$

Stein constrains the solution such that $r \le i$ and calls this the "No free lunch constraint." Therefore, using the stochastic calculus in Model I is the first term in (10):

$$dP(t) = P(t) \left(\alpha(t)dt + epdwp(t) \right)$$

$$dP(t)/P(t) = \left(r - \alpha y(t) + \frac{1}{2}\sigma^2_p \right) dt + \sigma_p dw_p$$
(10)

where $\alpha(t)$ represents the asset's drift component and the interest rate is represented by the sum of i and a Brownian Motion term as follows:

$$i(t) = idt + eidwi(t)$$

Substituting (10) in (7) and derive (11), the optimal debt ratio in model I is as follows:

$$dP(t)/P(t) = \left(r - \alpha y(t) + \frac{1}{2}\sigma^2_p\right)dt + \sigma_p dw_p \tag{11}$$

Consider $\beta(t)$ as deterministic.

Model II

In model II, the price equation is (12). The drift is $\alpha(t)dt = \pi dt$ and the diffusion is *pdwp*.

$$dP(t)/P(t) \quad \pi dt + \sigma_p dw_p \tag{12}$$

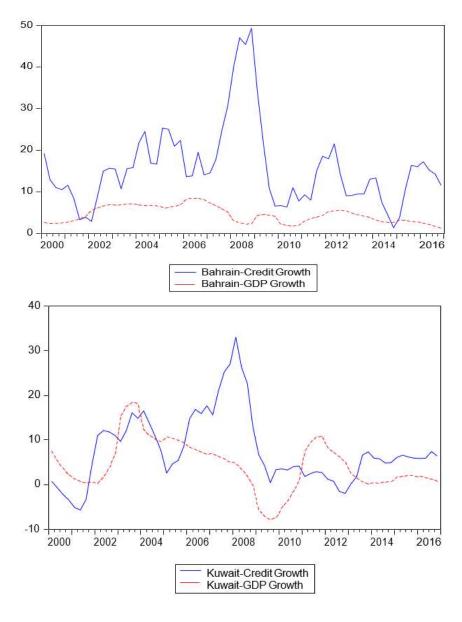
The optimal debt ratio f*(t) is (13). Consider $\beta(t)$ as deterministic.

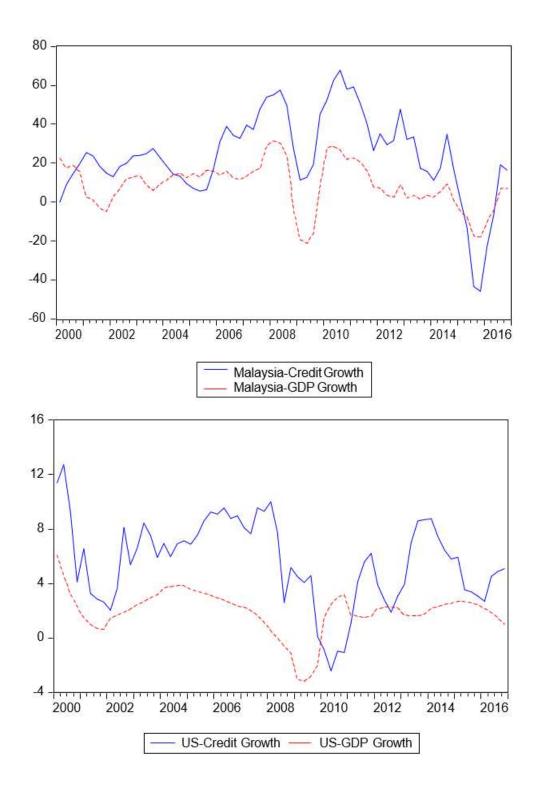
$$f^*(t) = \left[(\pi) + \beta(t) - i \right) - \frac{\left(\frac{1}{2}\right)(\sigma_p^2 - \sigma_i \sigma_p \rho)}{\sigma^2} \right]$$
s.t.
$$\sigma^2 = \sigma_i^2 + \sigma_p^2 - (2\rho_{ip}\sigma_i \sigma_p)$$
(13)

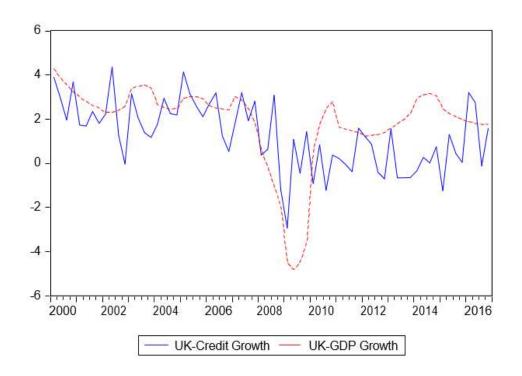
In terms of a maximization portfolio decision we have:

$$\max_{a_t} \left[\alpha_t \left(E(R_{t+1}) - R_{F,t+1} \right) - \frac{k}{3} \alpha_t^2 \sigma_t^2 \right] \tag{14}$$

APPENDIX B AVERAGE QUARTERLY GDP AND CREDIT GROWTH FROM 2000 TO 2016







APPENDIX C CORRELATIONS

			Leverage	Credit Growth	GDP Growth
Kendall' s tau_b	Leverage	Correlation Coefficient Sig. (2-tailed) N	1.000	001	102**
			.	.980	.005
			34 0	340	340
	Credit Growth	Correlation Coefficient Sig. (2- tailed) N	001	1.000	.359**
			.980	ŀ	.000
			340	340	340
	GDP Growth	Correlation Coefficient Sig. (2-tailed) N	102 ^{**}	.359**	1.000
			.005	.000	ŀ
			340	340	340
Spearman's rho	Leverage	Correlation Coefficient Sig. (2-tailed) N	1.000	.008	145**
				.887	.008
			340	340	340

Credit Growth Correlation Coefficient 0.008 1.000 .51 Sig. (2-tailed) N .887 . .000	13**
340 340 340	1
Sig. (2-tailed)	000
N .008 .000 . 340 340 340	40