APPLYING THE FERMI ESTIMATION TECHNIQUE TO BUSINESS PROBLEMS

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It is often necessary to make quick estimates when neither time nor resources are available for making traditional assessments. This is particularly true at the idea stage of product development when even a gross estimate could be useful for heading off ill-advised expenditures. The Fermi question, with which the scientific and engineering community has long been comfortable, is a helpful starting point for gaining insight into order of magnitude estimation. Although numerous worked-out solutions to Fermi questions are available, a systematic approach to solving them is not. This led the authors to develop a methodology which could be easily implemented in a traditional business course. A more pragmatic reason for introducing Fermi questions to business students is that corporations now employ these questions in the job interview process, as a means of gauging applicants' analytical skills. On this basis alone, business students should be taught the methodology, as it may immediately have relevance to furthering their careers.

INTRODUCTION

In business, it is often necessary to make quick estimates when neither time nor resources are available for making traditional assessments. For example, this is particularly true at the idea stage of product development when experiential data specific to the commercial viability of the idea does not yet exist, yet deciding whether to continue investing in the idea cannot be avoided. At this juncture, even a gross estimate is very useful to head off ill-advised expenditures, which are unlikely to generate a baseline profit. A back-of-the-envelope determination of market size, costs, or technical feasibility may be needed. Such a calculation ignores details, focuses only on major factors, and aims at an estimate within an order of magnitude (a power of 10) of the actual answer. Whether or not to proceed to the next level of development can then at least be based upon an educated guess of the product's potential.

A quick estimation process, with which the scientific and engineering community has long been comfortable for order of magnitude estimation, is the Fermi question (Orzel, 2007). Order of magnitude estimation seeks an estimate at least as accurate as the correct answer rounded to the nearest power of ten. In other words, if the correct answer is actually 40 million, an estimate as low as 10 million or as high as 100 million would be within the range of an acceptable estimate. A Fermi question is the name given to questions that can be answered, by an educated estimation process that relies on making reasonable assumptions, rather than on researching related information. A typical Fermi question would be, “How many shopping malls are there in the United States?” Fermi questions require several steps of estimation, and the estimator must begin by identifying the data values or factors, which would be needed to calculate the desired answer and estimate them.

So, in the case of pizza sales in New Jersey, useful factors might be: the population of NJ; average pizza consumption per person per week, number of pizza places in NJ, etc. During the process of estimating factors, errors tend to cancel each other out, with some overly high estimates being counterbalanced by overly low estimates. After estimating these factors, the desired answer can quite often be calculated, with surprising accuracy, i.e., within a factor of ten of the correct answer.

Until recently, business educators interested in teaching students to use Fermi questions would find it difficult to locate published questions related to the business world. Fermi questions and their solutions tended to involve physics, chemistry, biology, and other so-called hard sciences (Science_Olympics, 2009). However, this may be changing as the trend is for consulting firms and corporations such as Google and Microsoft to ask applicants to try to answer Fermi questions, as part of the job interview process, with the goal of identifying creative thinkers (Levin, 2007; Nakov, 2007) (W. Werbel, personal communication, Feb 17, 2008); so, it seems the time is ripe for greater focus on the application of the Fermi technique to business. The authors have identified a number of Fermi questions, some of which were used on actual job interviews, and provide them so that business educators can incorporate them into classes in areas such as marketing, economics, and entrepreneurship.

Also, to facilitate the use of Fermi questions in business courses, the authors describe a methodology for simplifying their solution. Although numerous worked-out solutions to problems are available, a systematic approach to solving them is not. Business educators may use the methodology to produce a template for students to follow, which functions as a learning tool.

HISTORY AND BACKGROUND

The eponymous Fermi question owes its origin to Enrico Fermi (1901-1954), an Italian physicist best known for his contributions to nuclear physics and the development of quantum theory. Fermi was awarded the 1938 Nobel Prize for physics for his work on the nuclear process. Shortly thereafter, Fermi was forced to flee Italy to escape the Fascist regime of Benito Mussolini and settled in the United States, first at Columbia University and then later at the University of Chicago. During World War II, he was a member of the Manhattan project team that developed the atomic bomb in Los Alamos, New Mexico (Verma, 2006; Morrison, 1963).

Fermi was legendary for being able to make a numerical estimate using information that at first blush seemed too meager to produce a quantitative result. His process of successive approximations "zeroed in" on answers by identifying that the value in question was certainly larger than a particular number and less than some other amount. He would proceed through a problem in that fashion and, in the end, have a quantified answer within identified limits (Derry, 2002). Today, Fermi questions have become a staple of high school science competitions and also appear in many college science texts. Popular Fermi questions are: How many gallons of gasoline are used each year in France? How many hot dogs will be eaten at major league baseball games during a one-year season? What is the weight of solid garbage thrown away by American families every year? What is the total length of hair on the average woman's head? (Brookes, 2006; Weinstein & Adam, 2008).

A CLASSIC EXAMPLE

Let's begin by solving a classic Fermi question: How many piano tuners are listed in the San Francisco phonebook? At first glance it may not seem possible to arrive at a solution without simply guessing, but by making reasonable assumptions, one can make surprisingly accurate estimates. Let's see how it might be done:

- estimate the population of San Francisco at two million;
- assume that families own pianos;
- estimate five persons per family which yields 400,000 families in San Francisco;
- estimate that one out of ten families have pianos, which yields 40,000 pianos in the city;
- estimate that pianos are tuned once per year which yields 40,000 pianos tuned each year;
- estimate that each tuner can tune four pianos per day, 200 days per year which yields 800 pianos per tuner per year;

or 40,000 pianos/800 tuned per tuner = 50 piano tuners.

Checking online, we find that 46 piano tuners are listed for San Francisco (yellowpages.com, 2009). At any estimation step above, we could have chosen a different number, e.g., we could have estimated the population to be 1 million, but this would still yield a reasonably accurate estimate. In fact, the more assumptions and estimates made, the more estimation errors tend to cancel. Interestingly, researchers have found that the average of two guesses is more accurate than either guess alone (Vul & Pashler, 2008). So, using more than one estimator and averaging their estimates should lead to greater accuracy.

CHALLENGES IN TEACHING FERMI ESTIMATION

The authors, who share both business and science backgrounds, introduced estimation techniques in a course they taught on invention to undergraduate students of varying majors. The goals of the course were for student teams to come up with an invention, which could be prototyped within one semester, present a comprehensive marketing plan for its development, and become familiar with intellectual property law basics. At times the invention course was offered through the school of science and at other times through the school of business. Students' majors included physics, nursing, psychology, business, and computer science. There was no prerequisite for the course.

Fermi questions were presented early in the course, as soon as students had selected an invention idea to develop, and provided a segue to a course module on how to market their invention. Students were divided into groups of four and five and as an in-class exercise, asked to estimate the number of piano tuners listed in the San Francisco phonebook. They were instructed not to use the Internet or any external sources of information.

At first students seemed unsure about how to begin solving the problem and incredulous that they would be able to develop an accurate answer. With some prompting on how to proceed, however, almost all student groups generated an estimate within one order of magnitude of the correct answer. Of course, depending on the approach selected, they arrived at the answer through estimating a variety of different factors. The solution, as shown above, was thoroughly discussed and a handout provided.

This in-class activity was so successful that the students were given the homework assignment of estimating the amount of water going over Niagara Falls each day. Students were instructed to show their factor estimates and explain how they reached their results. Surprisingly, very few students seemed to have absorbed an understanding of the method for solving a Fermi question. Most simply guessed at the answer. Each of the five times the class was offered, we observed the same result.

Undeniably, there is some complexity inherent in identifying the factors to use in the above problem, in which velocity of water, volume of water, and other issues affect the desired answer, the amount of water going over the falls each day. While providing examples is useful in communicating problem solution, we have found that the typical student, despite understanding the examples, still resorts to guessing when presented with a new problem. Students don't seem to know where to begin when presented with a problem, which is different from the one which has been explicated. While a literature search revealed many examples of solved Fermi questions and the assumptions used, which led to their solution (Charity, 2003; Weinstein & Adam, 2008) no standardized guide to the solution process is available. Accordingly, the authors propose the following model as an aid to students.

THE ROLE OF FACTORS

In making estimates of a desired quantity, certain bits of information are needed, which we call "factors." A familiar business metaphor would be Return on Assets, (net income/total assets) which is determined from the product of the two factors Profit Margin (net income/sales) and Asset Turnover (sales/total assets) (Collins & Devanna, 1990). That is:

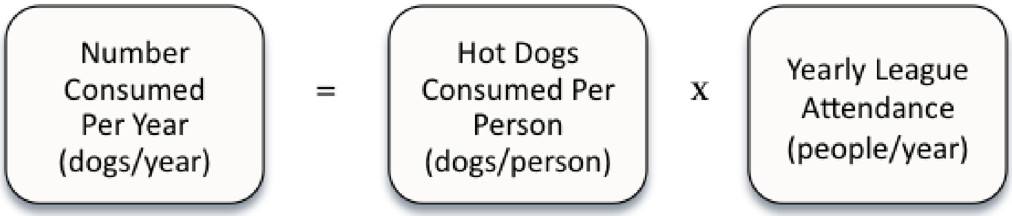
$$\frac{\text{Net Income}}{\text{Total Assets}} = \frac{\text{Net Income}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Total Assets}}$$

Some factors may be readily known or ascertained, while others must be further broken down into more fundamental factors. However, in the case of a typical Fermi problem, identifying these factors is not necessarily straightforward and requires some thought.

AN EXAMPLE

Let's examine the Fermi question: How many hotdogs are consumed at major league baseball (MLB) games each season? The first step is to determine at least two factors that will yield the number of hotdogs consumed. That is, the number of hotdogs consumed per year is simply the product of the two factors: *Yearly League Attendance* and *Hotdogs Consumed per Person*. This is shown graphically in Figure 1, where each factor is shown as a block.

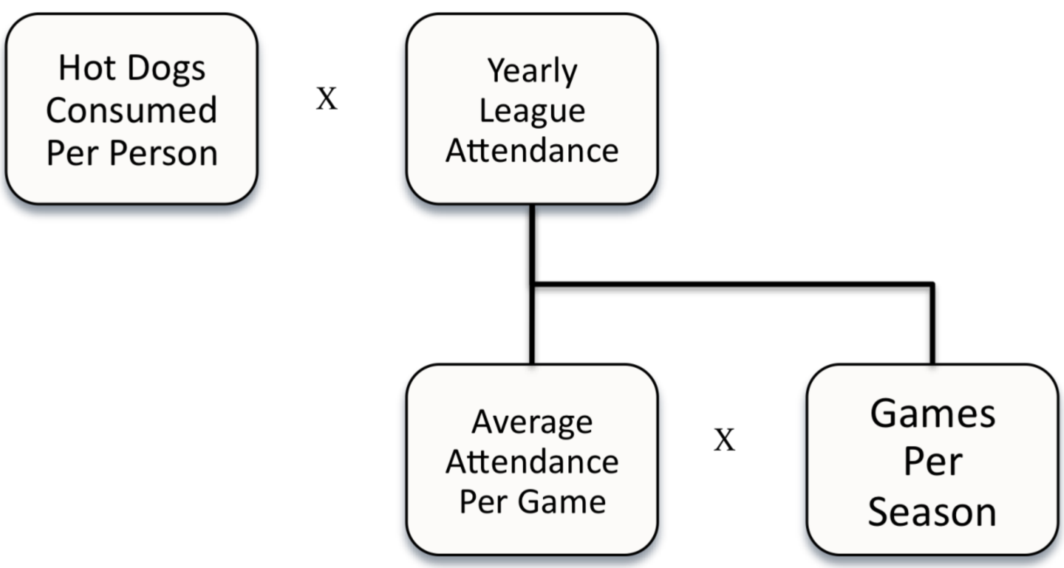
Figure 1



If during the process, any factor is known or can be easily ascertained, that value should be used (we do not allow this during a class assignment). Here, we will assume that we know little or nothing about major league baseball and all factors must be estimated and not researched.

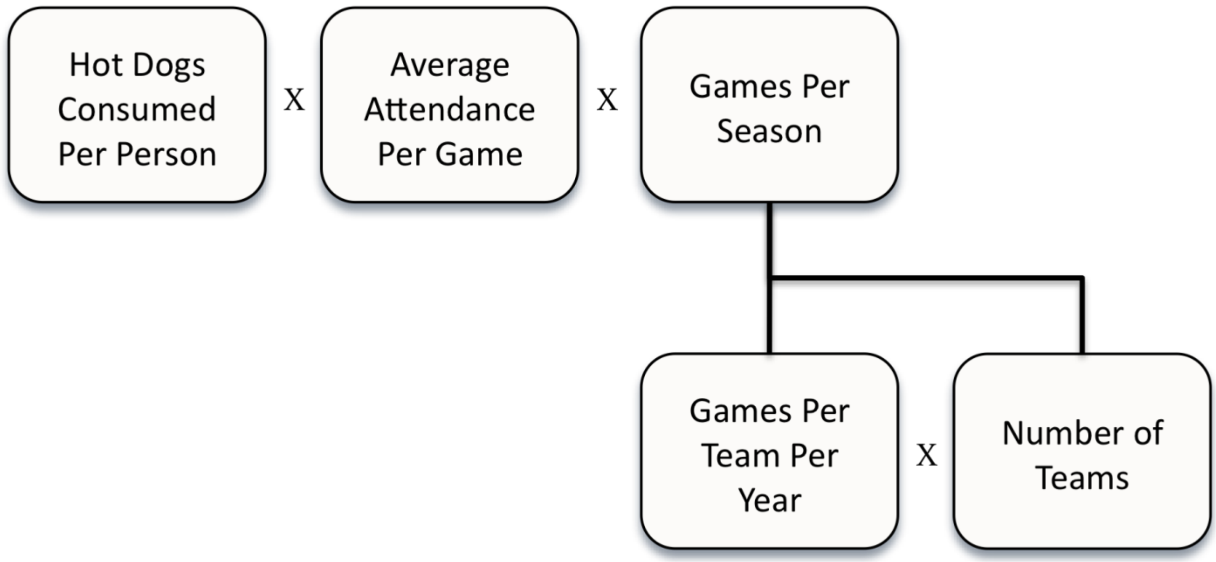
The next step is to determine which of the two above factors can be reasonably estimated and which cannot. While *Hotdogs Consumed Per Person* can be reasonably estimated, *Yearly League Attendance* cannot, without additional information. *Yearly League Attendance* could be further broken down into the factors of *Average Attendance Per Game* and *Games Per Season*, as shown in Figure 2.

Figure 2



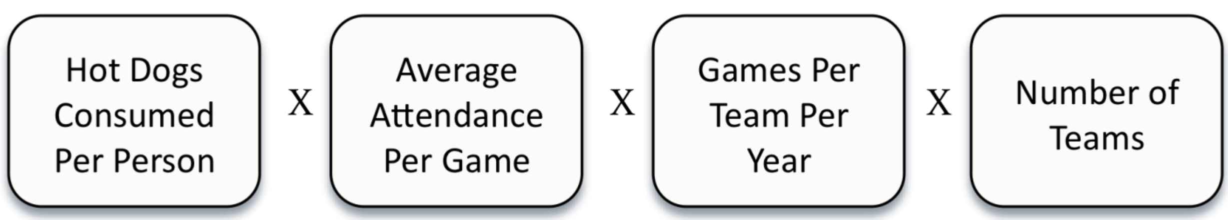
Now we need to repeat the above process of determining if any factor still needs to be broken down into additional factors. In this case, *Average Attendance Per Game* may be reasonably estimated, but *Games Per Season* cannot without additional information. This latter term could be estimated if we knew *Games Per Team Per Year* and *Number of Teams* (Figure 3).

Figure 3



Both of these latter terms can be reasonably estimated without further information. (The average person has had some exposure to eating hotdogs and even a cursory familiarity with any professional sport would provide enough information to make an estimate, albeit a grossly inaccurate one.) So, the number of hotdogs consumed at MLB games in a season is therefore a product of the following quantities that must each be estimated: (*Hotdogs consumed per person*); (*Average attendance per game*); (*Games per team per year*); and (*Number of teams*), as shown in Figure 4.

Figure 4



We calculate each factor in turn and start with hotdogs consumed per person. We might initially estimate that the average person would eat less than one hotdog per game. But one hotdog per person still seems a little high, so we will use one hot dog for every 3 people, or 1/3 as our upper limit of hotdogs per person. Now we select a lower limit, which is an order of magnitude lower than our upper limit, or 1/30. We'll use the average of these two numbers to calculate our factor estimate of: 0.18, or about one hotdog per every seven people.

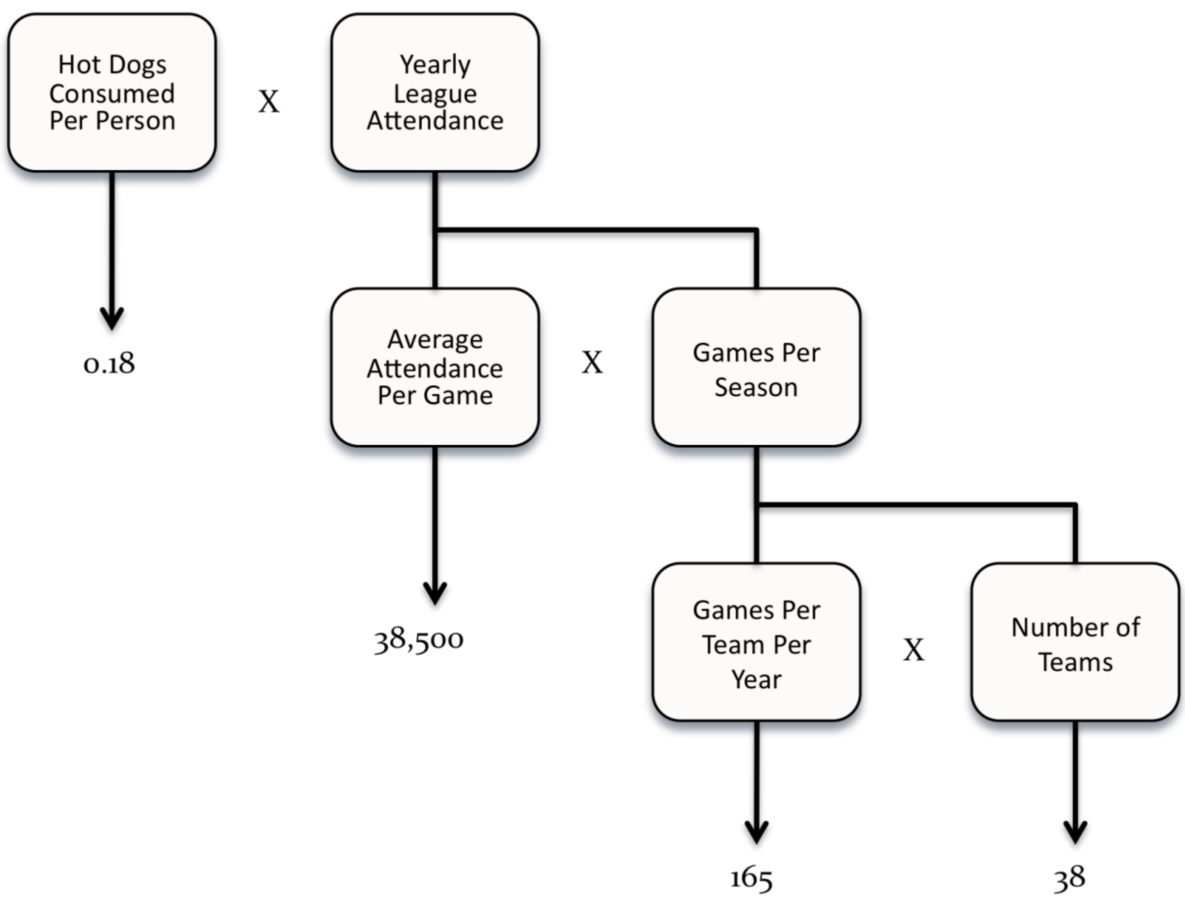
Our next factor is average attendance per game. We certainly know that average attendance per game is greater than 100 and less than 1 million, but we can be more precise. Certainly greater than 10,000 and less than 100,000 seems excessive, but a true baseball neophyte estimator might actually be more comfortable with this range. We will try a slightly tighter range where the upper limit is 70,000 and the lower limit is 7,000. The average of these two numbers is 38,500, which seems reasonable.

Looking at games played per year, if we assume that the baseball season is long and games are played often, we might choose 300 games per year as an upper limit, which gives us 30 as the lower limit. The average of these two numbers yields 165 as our estimate for the number of games played by per team per year.

Lastly, in estimating the number of teams, there are probably fewer than 100 and more than 10 teams, but again we can probably assume there are fewer than 70 teams. This sets our lower limit as 7 yielding 38 as our estimate of the number of teams., but for our calculation, we will use 38.

Accordingly, as shown in Figure 5, we have the following calculation: 0.18 hot dog/fan x 38,500 fans/game x 165 games/team x 38 teams = 43 million hotdogs/year consumed by fans at MLB games. However, we have actually double counted, as any game has two teams playing at a time; so our estimate is 22 million hotdogs/year. Even given that another estimator might use different estimates for each factor, results would be similar due to variations in individual estimates tending to cancel one other.

Figure 5



Actually, according to ESPN, average attendance at MLB teams in 2004 was 30,400 and there were 30 teams, each playing 161 games (ESPN.com, 2009). The number of hot dogs consumed by fans at MLB games for the 2004 season was 26 million, according to the National Hot Dog & Sausage Council (Newman, 2004). Based upon these numbers, the average number of hot dogs consumed per fan is 0.18 or one hot dog per 5.5 fans. The comparison between our estimated results and actual figures are shown in Table 1. As demonstrated, when the methodology is followed, Fermi questions can invariably be solved to provide an answer within one order of magnitude of the correct answer. Again, in estimating any factor above, we could have chosen different numbers, but still have arrived at a reasonably accurate estimate.

TABLE 1
COMPARISON OF OUR ESTIMATED RESULTS AND ACTUAL FIGURES

	actual	**our estimate**
hotdogs/person	0.18	0.18
attendance/game	30,400	38,500
games/team	161	165
number of teams	30	38
hotdogs consumed	26 million	22 million

FERMI QUESTIONS AND JOB INTERVIEWS

In addition to serving as a valuable business tool for providing rough estimates of potential market size, Fermi questions also are more immediately important to business students because they are sometimes an important part of a job interview, especially for MBA graduates (Levin, 2007). This phenomenon seems to be occurring with greater frequency as indicated by a recent video by vault.com, a career and job information website (vault.com, 2009). The video provides business students with a reasonable idea of what to expect when a Fermi question is part of a job interview. The question posed in the video is “how much money is spent on haircuts in the US in one year?” The would-be job applicant first demonstrates incorrect approaches to answering such as blurting out a number, asking for the interviewer’s calculator, and spending more than ten minutes on calculating the answer. The video then presents a reasonable, step-by-step approach to estimating the answer, which is not as detailed as the methodology presented in this article. Ivy league undergraduates also report being asked the Fermi questions on job interviews; some of these appear below along with the type of company or department, where the interview took place:

- How many cows does McDonald's slaughter each year in the US to make hamburgers? - Corporate Strategy
- How many pieces of luggage go through JFK airport in a year? - Consulting
- How many Starbucks coffee shops are there on the island of Manhattan? - Consulting
- How many pounds of turkey are eaten in the US on Thanksgiving day? - Healthcare Consulting
- Estimate the market for day care centers in a town of 200,000 in upstate New York - Consulting
- How many videos does Blockbuster rent out in a year? - Corporate Strategy

CONCLUSION

The business curriculum is inherently a quantitative course of study with its emphasis on statistics, economics, finance, accounting, and operations research and the tools of business are directed at identifying a bottom line, though the definition of “bottom line” has been expanded of late (Savits & Weber, 2006). By borrowing a quantitative tool from the world of science, the Fermi question, any for-profit institution can gain a better idea of the potential for its innovations to affect its bottom line. Accordingly, business students should be encouraged to gain experience solving Fermi questions and the methodology described in this paper should enhance the teaching of this most-useful method of estimation.

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