

## **The Tax Structure of Interest Rates and the Darby Effect – An Application of the M-TAR Error Correction Model**

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*Recent increases in both federal/states budget deficits, expected tax/inflation hikes, and their impact on Treasury and municipal bond markets, have renewed interest in the reported Darby effect in the US tax structure of interest rates. The controversial evidence of a Darby effect in the tax spread has mainly been explored under the linearity presumption. Although linearity is an appropriate starting point, it is incapable of capturing the true underlying dynamic adjustments. This paper presents empirical evidence indicating that the time series variable depicting the tax structure of interest rates is inherently non-linear. The apparent non-linearity has profound implications for the reported Darby effect, as well as for the decreasing tax structure of relative yield spreads. The two widely used non-linear models TAR (threshold auto-regressive) and M-TAR (momentum TAR) have been considered. The findings suggest that the M-TAR model is more capable of capturing the tax spread's non-linear dynamic adjustment process, though it does not depict any evidence of a Darby effect in the sample period 1952:q2-2008:q4.*

### **INTRODUCTION**

The tax structure of interest rates measured by the discrepancy between yields to maturity of taxable and tax-exempt bonds (the tax spread), has predominantly been used in two major lines of macroeconomic research. First, for the much debated Darby effect - a logical extension of the renowned Fisher effect in which Fisher (1930) envisions a one-on-one relation between nominal interest rates and the expected rate of inflation so that the expected real interest rate remains constant. The necessary conditions for the Fisher effect to hold are a relatively low inflation rate and no interest income tax differentials. However, as Fama (1975), Darby (1975), and Feldstein (1976) among others argue, due to higher inflation rates and the U.S. income tax structure, nominal interest rates should increase more than the expected inflation rate – a phenomenon that has become known as the Darby effect.<sup>1</sup>

The empirical testing of the Darby effect has been done by regressing different measures of taxable nominal interest rates (e.g., interest rates on government or corporate bonds) on proxies for the expected rate of inflation and a vector of related explanatory variables. While the intercept is a measure of the expected real interest rate, the regression coefficient of the expected inflation rate should be significantly greater than one in order to confirm the existence of a Darby

effect. The reported evidence in pages of high ranked journals in the late 1970s and early 1980s is generally supportive of even less than the Fisher effect. However, Ayanian (1983) purportedly puts the issue to rest by reporting “unmistakable” evidence of a Darby effect in the U.S. financial markets.<sup>2</sup> The novelty of his approach is that he considers the municipal bond rate (MBR) as a proxy for the expected after tax real interest rate and the expected rate of inflation (both of which are difficult to measure accurately), while the Treasury bill rate (TBR) is a measure of the nominal interest rate. Then, in the context of a simple linear regression model, he argues that “The growing yield spread between taxable and tax exempt bonds accompanying the rising nominal yield on tax-exempt bonds is evidence of the Darby effect,” (Ayanian, 1983, p. 764).<sup>3</sup> Secondly, there has been a great deal of empirical work on the determinants of relative tax spreads – to name a few, see Buser and Hess (1986), Green (1993), and Hein & Mercer (1990). In particular, Kryzanowski et al. (1995) using monthly observations 1954:2-1987:12, empirically demonstrates that the term structure of relative tax spreads  $[(TBR - MBR)/TBR]$  is inversely determined by the implied tax rate imposed on taxable bonds, the default risk premia imposed on municipal bonds, the tax-timing option values, and by the rate of inflation. Moreover, the implied tax rate increases as the maturity of taxable bonds increases, the risk premia and the tax-timing option values tend to increase as maturity expands, but the response of relative tax spreads to inflation is generally mixed.<sup>4</sup> In a sense, as the economic theory suggests, the right-hand-side variables of Kryzanowski’s regression model significantly contribute to the falling term structure of relative tax spreads<sup>5</sup>.

In the Darby studies, the presumption is that TBR and MBR are stationary and that they adjust to the tax spread linearly. Thus, Ordinary Least Squares (OLS) has been used for the empirical investigation. Although Kryzanowski utilizes the maximum likelihood (ML) procedure, the underlying hypothesis is that the relative tax spread’s dynamic adjustment is symmetric.<sup>6</sup> However, since the tax spread is pro-cyclical and sensitive to institutional changes, the likelihood of asymmetric adjustments in the above studies is noticeably high. Consequently, it is *prima facie* that the degree at which TBR (or MBR) changes relative to the tax spread is determined by its threshold(s). Moreover, even if the spread behaves linearly, there is really no reason to believe that its dynamic adjustments are also linear. Accordingly, at times, the tax spread may not increase enough (i.e.,  $TBR > MBR$ ) when MBR is rising to fully compensate for the investors’ possible “fiscal illusion” if the Darby effect is to be expected. In like manner, without taking into account the apparent regime switching, the term structure of relative tax spreads may not change sufficiently to accurately reflect the implied taxes and default risk premia on Treasury and municipal bonds, respectively. In essence, without incorporating the identifiable thresholds, the asymmetric behavior of the tax spread in the aforementioned studies, renders empirical evidence that is quite unsettled. Since the tax spread along with most key economic variables (e.g., GDP, industrial production - IP, and the unemployment rate - UN) tend to behave non-linearly during phases of a business cycle, the macroeconomic literature on the implications of non-linear time series has recently grown inordinately. Indeed, the asymmetry of GDP, IP, and UN during a typical business fluctuation has been confirmed by Neftci (1984), Terasvirta & Anderson (1992), Porter (1995), and Balke & Fomby (1996) among others.<sup>7</sup> However, more recently, the focus appears to have been shifted from merely demonstrating asymmetric behavior of the above time series variables to pinpointing the exact nature of such asymmetry – see for example, Sichel (1993). Accordingly, the idea is that not only are the dynamic adjustments of these variables asymmetric, but also that we may observe “sharpness” or “deepness” incorporated in them. Generally speaking, sharpness implies a situation where a

contraction is longer than an expansion and deepness is indicative of a more prolonged trough than peak.

Prior to investigating nonlinear dynamic adjustments, an important inquiry is whether or not there is a long-run equilibrium (attractor) toward which the above time series variables move. Unfortunately, it is impossible to utilize the commonly used integration tests because the majority of them are based on the linearity presumption. The presumed linearity might be a good approximation but as mentioned before, it is incapable of capturing the true dynamic adjustments of most time series variables. In fact, the class of models dubbed as TAR (threshold autoregressive) proposed by Tong (1983) and M-TAR (momentum TAR) suggested by Enders and Granger (1998) are only two examples of non-linear adjustments. In the TAR type models, the autoregressive parameters decay over time (deepness), whereas in the M-TAR class of models, the decaying process has a clear tendency towards one way or the other (sharpness). For example, having established asymmetric behaviors in the term spread (10-year government bond rate minus 4-to-6 month federal funds rate, 1958:q1–1994:q1), Enders & Granger demonstrate that it has an “attractor,” a threshold, and that it is a good candidate for the M-TAR model. Furthermore, the implied non-linear error correction model depicts a much different dynamic adjustment process than its linear (symmetric) counterpart. The up-shot of their findings is that the linear integration tests unduly masks the governing mechanisms built into the interest rate differential, which results in misleading long-term dynamic adjustments.

This paper applies both TAR and M-TAR models to the tax spread ( $SPR = TBR - MBR$ ) in the sample period 1953:q2-2008:q4. Compared to the term spread, there are at least three reasons as to why such an application may be of interest. First, in the presence of recent unprecedented federal and state budget deficits and their impact on interest rates, the reported evidence of a symmetric Darby effect in the tax structure of interest rates can be re-examined under the non-stationarity and non-linearity (asymmetry) proposition. Second, regarding the business cycle's sharpness/deepness, unlike the countercyclical nature of the term structure of interest rates, SPR closely follows the cycles.<sup>8</sup> During the expansionary phase of a business cycle, the percent rate of growth of real GDP (income) rises, which increases interest rates regardless of their tax status. Given the progressive nature of income tax rates in the US, higher incomes result in higher tax burden (brackets) and thus, the demand for tax-exempt bonds (e.g., municipal bonds) increases, resulting in a higher price (lower yields) of these bonds. For taxable bonds (e.g., Treasury bonds), the expansionary period implies a lower demand, resulting in a lower price, and higher yields. In short, a rising taxable interest rate coupled with a falling tax-exempt rate, should increase the tax spread during an up-turn and decrease it during a downturn. Third, unlike the term spread, the tax spread is not subject to a time varying risk differential because both interest rates are of the same term to maturity.<sup>9</sup> In that spirit, section two presents the TAR and M-TAR models of the tax spread followed by the findings in section three, and some concluding remarks are made in section four.

## **THE MODEL**

In the simplest scenario, the TAR model of the tax spread ( $SPR_t$ ) is of the following form:

**FIGURE 1  
THE TAR MODEL**

$$SPR_t = I_t [a + \sum b_i SPR_{t-i}] + (1 - I_t)[c + \sum d_i SPR_{t-i}] + e_t, \quad \text{and } i = 1, 2, 3, \dots, n.$$

where  $I_t$  is a dichotomous (1 or 0) variable,  $a$  &  $c$  are intercepts,  $b$  &  $d$  are the auto-regression coefficients, and  $e_t$  is a well behaved (white noise) error term. The nature of the dummy variable  $I_t$  (the indicator) is such that  $I_t = 1$  if  $SPR_{t-1} \geq \tau$  (threshold), and 0 otherwise. In essence, equation 1 is governed by two distinctive regimes; regime one states that when  $SPR_{t-1} \geq \tau$ ,  $I_t = 1$ ,  $(1 - I_t) = 0$ , and the applicable autoregressive model is  $[a + \sum b_i SPR_{t-i}]$ . However, under regime two,  $SPR_{t-1} < \tau$ ,  $I_t = 0$ , and  $(1 - I_t) = 1$ , then the relevant model would be  $[c + \sum d_i SPR_{t-i}]$ . In short, equation (1) switches between the two models intermittently based on the regime under which it operates. In an unlikely case where the two regimes operate analogously with no discernable difference [say,  $I_t = 1$  and  $(1 - I_t) = 0$ ], equation (1) reverts back to a simple autoregressive model of the form:

**FIGURE 2  
THE AUTOREGRESSIVE MODEL**

$$SPR_t = a + \sum b_i SPR_{t-i} + e_t, \quad \text{and } i = 1, 2, 3, \dots, n.$$

Assuming  $i = 1$ , subtracting  $SPR_{t-1}$  from both sides and rearranging would result in the familiar Dickey Fuller-DF (1979) linear equation, which is commonly used for integration tests as follows:

**FIGURE 3  
THE D-F MODEL**

$$\Delta SPR_t = a + b_1 SPR_{t-1} + e_t, \quad \text{and} \quad b_1 = (b - 1),$$

where  $\Delta$  is the first differencing operator. Of course, equation (3) can be generalized or augmented with the lagged values of the dependent variable if the residual autocorrelation persists.

In the absence of asymmetry, equation (3) can be used for integration tests and after accepting the null hypothesis of a unit-root, the underlying variables (TBR & MBR) can be further tested for co-integration. If co-integration is confirmed, then in a search for the Darby effect, both TBR and MBR can be examined in the context of a symmetric error correction model (SECM). However, if the tax spread time series is asymmetric, the SECM is not only inapplicable, but also misleading. Consequently, it is imperative to subject equation (3) to non-linearity tests and upon confirmation, the type of asymmetry (TAR or M-TAR) can be determined as is reported below:

**FIGURE 4  
THE TAR OR M-TAR MODEL**

$$\Delta SPR_t = \rho_1 I_t [SPR_{t-1} - \tau] + \rho_2 (1 - I_t)[SPR_{t-1} - \tau] + e_{1t}.$$

where  $\rho_1$  &  $\rho_2$  are the auto-regression coefficients depicting the speed at which  $SPR_t$  adjusts to its long-run equilibrium value (given the threshold,  $\tau$ ). The specified indicators are  $I = 1$  if the one period lagged spread,  $SPR_{t-1} \geq \tau$ , & 0 otherwise ( $SPR_{t-1} < \tau$ ) for TAR, and  $I = 1$  if  $\Delta SPR_{t-1} \geq \tau$ , & 0 otherwise ( $\Delta SPR_{t-1} < \tau$ ) for M-TAR. Assuming the existence of an attractor by rejecting the null hypothesis that  $\rho_1 = \rho_2 = 0$  (using the critical values), then further rejecting  $\rho_1 = \rho_2$  (based on the standard F-test), is indicative of asymmetric adjustments. Subsequently, the asymmetric error correction model (AECM) is a logical generalization of equation (4) by way of incorporating appropriate lagged values of both the dependent and independent variables.

The numerical value of  $\tau$  (or band thresholds if applicable) would have to be estimated in the same way as the numerical values of  $\rho_1$  &  $\rho_2$ . A consistent estimate of  $\tau$  has been obtained in accordance with the procedure explicated by Chan (1993). The Chan approach precludes  $\pm 15\%$  of the observations and also ranks them in an ascending fashion. Moreover, using OLS, equation (1) is estimated recursively within the  $\pm 15\%$  constraint. The estimated model whose residual sum of squared is minimal produces a consistent estimate of  $\tau$ , which can be used to estimate equation (4) appropriately.<sup>10</sup>

## EMPIRICAL FINDINGS

Equation (3) has been used to identify the best fitting version of the D-F model for integration tests. Using the Schwarz Bayesian criterion (SBC) and 14-period initial lags, the following model is deemed appropriate:<sup>11</sup>

### FIGURE 5 THE ESTIMATED D-F MODEL

$$\Delta SPR_t = 0.038 - 0.045 SPR_{t-1}$$

(1.52) (-2.17)

SBC = 0.126 ARCHm-1 = 24.85 (probability = 0, arch order = 4)  
Q-statistic = 2.30 (probability = 0.15, autocorrelation order = 1).

where numbers in the parentheses are t-statistics, ARCHm-1 is the McLeod-Li (1983) autoregressive conditional heteroscedasticity portmanteau test for the existence of non-linearity, and Q is the estimated chi-square for the Breusch-Godfrey autocorrelation test. As can be seen, the estimated ARCHm-1 resoundingly rejects the null hypothesis of linearity, which necessitates further testing for the presence of a threshold. Using Mackinnon's (1996) critical value of -2.87, the null hypothesis of a unit root is also accepted at the 5% significance level.

For the TAR version of equation (4), Chan's approach estimates the threshold  $\tau = 1.68$  and for the M-TAR version,  $\tau = 0.55$  - the findings are reported below:

### FIGURE 6 THE ESTIMATED TAR MODEL

**TAR** →  $\Delta SPR_t = -0.11 I_t[SPR_{t-1} - 1.68] + -0.01 (1 - I_t)[SPR_{t-1} - 1.68] + e_{1t}$

(-2.06) (-1.22)

The Akaike information criterion (AIC) = 0.094 SBC = 0.125





because capital gains were generally exempted from taxes. The tax reform of 1986 decreased the tax differential considerably, though the Darby effect may still exist due to the fact that taxes on capital gains are deferred until they are realized. Therefore, the effective tax rate on capital gains is below the normal tax rate applicable to interest incomes. The Darby effect may be present in the long-run in the absence of such a tax differential if the supply of saving is believed to be sensitive to the real after tax return to capital.

2. Standard macroeconomic textbooks referred to the reported findings as “settling the Darby controversy” – see for example, Darby and Melvin, 1986, p. 85.
3. If  $T$  is the marginal tax rate, yields on 1-year Treasury and prime grade municipal bonds are  $TBR_1$  and  $MBR_1$  respectively, then the arbitrage equilibrium is  $(1-T) TBR_1 = MBR_1$ , or  $TBR_1 = 1/(1-T) MBR_1$ . The estimated significant regression coefficient (resulting from regressing  $TBR_1$  on an intercept and  $MBR_1$ ) reported by Ayanian in 1952:q1-1979:q4 turns out to be 1.63. The implication of the findings is that a 1% increase in  $MBR_1$  is associated with 1.63% increase in  $TBR_1$  and thus, capable of compensating marginal lenders in the 38.7% tax bracket. However, note that Ayanian’s model is subject to non-stationarity and non-linearity problems - the findings are spurious.
4. An exception is the 1979-1982 time period during which the intermediate target of the Fed was changed on Oct 6, 1979. The switch from the interest rate to monetary aggregate targeting resulted in a period of extreme interest rates volatility.
5. Unlike previous studies, Kryzanowski’s regression model has been estimated with bond maturities ranging from 1-30 years in a log-rolling fashion, i.e., 1-2, 1-5, 1-10, 1-20, 1-30, as well as 1-2, 2-5, 5-10, 10-20, and 20-30.
6. Generally, in large samples (asymptotic properties) OLS and ML produce the same results.
7. Note that there is also evidence to the contrary – see Falk (1986).
8. Since both the Fisher and Darby effects are by nature long-term phenomena, the tax spread is measured by the discrepancy between 20-year Treasury and general obligation municipal bond rates - the data are from the Federal Reserve Board (2009).
9. Municipal bonds are subject to de-facto default risks, but Treasury bonds are not.
10. For brevity, Chan’s procedure has not been discussed in depth – the detailed estimation and findings are available upon request.
11. Note that the augmentation term ( $\Delta SPR_{t-1}$ ) is not significant even at the 10% level.
12. The critical values are non-standard and have been reported by Enders (2004), Table F, panel (b).
13. Several different lag structures were examined - both AIC and SBC suggest the reported lag-lengths that are incorporated in equations (8) and (9), as well as those incorporated in the symmetric error correction models.
14. A linear co-integrating vector mandates significance for at least one of the estimated speed of adjustment coefficients (at the 5% level) in the TBR and MBR co-integrating space.
15. The estimated symmetric (linear) error correction models are (t-statistics are in parentheses and Resid = residuals from regressing TBR on an intercept and MBR):  

$$\Delta TBR_t = -0.03 \text{ Resid}_{t-1} + 0.92 \Delta MBR_t$$

t- value	(-1.36)	(18.17)
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Q-statistic(1<sup>st</sup>-order) = 1.67, Probability = 0.20      SBC = 0.123  
The Darby coefficient is 0.92 and significantly less than one at the 5% level.

$\Delta\text{MBR}_t = 0.08 \text{ Resid}_{t-1} + 0.01 \Delta\text{TBR}_{t-1} + 0.14 \Delta\text{TBR}_{t-2} + 0.28 \Delta\text{MBR}_{t-1} - 0.09 \Delta\text{MBR}_{t-2}$   
t-value    (2.89)                      (0.16)                      (1.59)                      (2.72)                      (-0.91)  
Q-statistic (1<sup>st</sup>-order) = 2.36, Probability = 0.12      SBC = 0.667.

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