

Determinants of Steady State Income: Is a Linear Specification Too Simple?

Kathleen E. Odell
Dominican University

This paper examines whether the Solow model of economic growth and steady state income is supported by available cross-section data. The paper argues that while income per worker is correlated positively with investment in capital (both physical and human) and negatively with population growth, a simple log-linear empirical model derived from the Cobb-Douglas production function oversimplifies these relationships. A nonlinear specification is presented as an alternative. Estimation of the nonlinear specification shows that for the period 1960-2000, the strongest marginal effects of education occur in countries where education levels are already relatively high.

INTRODUCTION

The economic growth literature contains numerous studies offering empirical support for the neoclassical growth model, but estimation in these studies is almost always conducted using a linear model and ordinary least squares (OLS) regression. However, the results of OLS estimation provide *average* coefficients, and these averages may or may not accurately represent the relationships in the data. There are two key implications of the assumption of linear coefficients. First, this assumption implies that all countries included in the cross-section follow identical linear growth processes. Second, the assumption implies that the coefficients on all explanatory variables are constant across *all levels* of those variables. Where income and growth are concerned, these are strong and testable implications. This paper argues that the simple log-linear model of steady state income is too simple. In particular, there is evidence of increasing returns to education in the cross-section data.

Empirical support for the neo-classical Solow (1956) model of economic growth was presented by Mankiw, et. al. (1992) (henceforth MRW). MRW used a log-linear estimation equation derived from the Cobb-Douglas production function. Their finding was that nearly 60% ($\bar{R}^2 = 0.59$) of the cross-country variation in income per worker could be explained by a “Textbook Solow Model” containing only the savings rate (as a proxy for investment in physical capital) and the population growth rate. Further, MRW developed an “Augmented Solow Model” which included both physical and human capital accumulation, and this augmented model explained nearly 80% ($\bar{R}^2 = 0.78$) of the variation in income per worker. Bernanke and Gurkaynak (2001) and Karras (2008) have determined that the Textbook Solow Model performs as well or better when additional data, available since the publication of the MRW paper, is included. Bernanke and Gurkaynak also re-estimate the Augmented Solow Model with updated data and find results similar to MRW.

Given the theoretical consistency, high explanatory power, and robustness of the log-linear model (estimated with OLS), it is tempting to conclude that this specification is an accurate description of the data. However, the importance of the linearity assumption is increasingly under scrutiny. As a result,

there now exists a substantial body of empirical growth research which supports the argument that a standard linear model (estimated with OLS) is inconsistent with the historical cross-sectional growth data. Both theoretical (Azariadis and Drazen (1990), Galor and Weil (2000)) and empirical (Durlauf and Johnson (1995), Hansen (2000), Liu and Stengos (1999)) work has suggested that a linear model is inconsistent with reasonable explanations of the economic growth process. However, few non-linear studies have been conducted using updated versions of the cross-sectional data. Further, there is little consensus in the literature about the most effective method for testing for nonlinearity.

This study demonstrates that for the period 1960-2000, the well-documented support for the Textbook Solow Model of growth is robust to the relaxation of the linearity assumption. However, the Augmented Solow Model, which adds the effect of human as well as physical capital to the model, is better described by a model that is nonlinear in the coefficient of human capital. This paper outlines a simple testing methodology for nonlinearity in the coefficients. The Generalized Additive Model (GAM) is a straightforward approach to performing a specification check on the linear specification. This methodology can be easily extended to test the specification of equations describing convergence in income; however, in this paper the testing is limited to a specification of the effect of savings, population growth, and education on steady state *levels* of income.

The analysis begins with re-estimation of MRW's basic empirical equations using OLS. OLS estimation is used to establish a baseline for comparison and to test whether updated data confirms findings from the original MRW study. Initial nonlinear testing is conducted using GAM together with the original MRW data on 98 countries for the period 1960-1985. Using the MRW data, both the population growth variable and the human capital accumulation variable are significantly nonlinear. This finding also appears using an updated version of the data for the 1960-1985 period.

When a longer period (1960-2000) is estimated using both OLS and GAM, the OLS results are qualitatively similar to the original MRW results. However, GAM specification testing suggests that the human capital accumulation variable is significantly nonlinear for this period, indicating that the linear specification over-simplifies the relationships between income and education. There is a notable disparity between the results for the 1960-2000 period and the results for the 1960-1985 period. For the 1960-1985 period, the growth rate of the working age population has a nonlinear effect on steady state income in both the Textbook and the Augmented Solow Models. This result is robust to the data set used for estimation. For the period 1960-2000, this nonlinear effect of population growth disappears. This result is surprising but supports Temple (2000), who observes that "we must wait until...the year 2005 to see whether empirical models originally estimated using data from 1960-1985 perform well over the 1985-2005 period" (p. 202). The results in this paper suggest major differences in the models describing these two periods.

The strongest finding of the paper is that the effect of human capital on income is nonlinear, increasing in the level of human capital. This result appears across different sets of countries, the two different time periods under study (1960-1985 and 1960-2000), and different measurement strategies for human capital.

As a test of the performance of the 1960-1985 models, these models are applied to the 1960-2000 data. With both linear and nonlinear models for the 1960-1985 period in hand, it is possible to compare the performance of these two models when applied to the extended updated data. The nonlinear model of steady state income estimated with GAM forecasts better out of sample than the linear model estimated with OLS. GAM reduces the sum of squared forecasting error by about 10% relative to OLS for both the Textbook and the Augmented Solow Models.

The rest of the paper is organized as follows. Section 2 outlines the empirical methodology, including the estimating equations for steady state income levels. Section 3 discusses data definitions and sources. The results of the OLS and GAM testing are presented in Section 4. The results of the out-of-sample forecasting are presented in Section 5. Section 6 offers conclusions and directions for further research.

EMPIRICAL METHODOLOGY

MRW study the effects of the growth determinants on standard of living. The basic OLS estimation in this paper follows the approach of MRW. For the Textbook Solow Model, assume a Cobb-Douglas production function, so that production at time t is given by the Equation 1.

EQUATION 1 COBB DOUGLAS PRODUCTION FUNCTION, TEXTBOOK MODEL

$$Y(t) = K(t)^\alpha (A(t)L(t))^{1-\alpha} \quad 0 < \alpha < 1$$

The notation is as follows: Y is output, K is physical capital, L is labor, and A is the level of technology. L and A are assumed to grow exogenously at rates n and g (respectively). In addition, capital depreciates at a constant rate δ . Assuming that a constant fraction of output, s , is saved, MRW show that log steady state income per capita will be given by the equation in Figure 2.

EQUATION 2 LOG STEADY STATE INCOME PER CAPITA, TEXTBOOK MODEL

$$\ln\left(\frac{Y(t)}{L(t)}\right) = a + \frac{\alpha}{1-\alpha} \ln(s) - \frac{\alpha}{1-\alpha} \ln(n + g + \delta)$$

Equation 3, the empirical equation used to estimate the model, follows from Equation 2.

EQUATION 3 EMPIRICAL ESTIMATING EQUATION, TEXTBOOK MODEL

$$\ln\left(\frac{Y(t)}{L(t)}\right)_i = \gamma_0 + \gamma_1 \ln(\bar{s}_i) + \gamma_2 \ln(\bar{n}_i + g + \delta) + \varepsilon_i$$

Here, \bar{s}_i is the savings rate for country i averaged over the time period under study, \bar{n}_i is the growth rate of the working age population for country i averaged over the time period under study, $g + \delta$ is assumed to be a constant 0.05, and ε is an error term assumed to be independent of s and n . The expected results are $\gamma_1 > 0$ and $\gamma_2 < 0$.

Similarly, MRW derive an empirical estimating equation for the *Augmented* Solow Model which splits capital into its physical and human components. The augmented model begins with the Cobb-Douglas production function, as outlined in Equation 4.

EQUATION 4 COBB DOUGLAS PRODUCTION FUNCTION, AUGMENTED MODEL

$$Y(t) = K(t)^\alpha H(t)^\beta (A(t)L(t))^{1-\alpha-\beta}$$

Here, $H(t)$ is the stock of human capital at time t and all other variables are defined as above. From the production function an equation similar to Equation 2 above can be derived, where s_k is the fraction of income invested in physical capital and s_h is the fraction of income invested in human capital.

EQUATION 5
LOG STEADY STATE INCOME PER CAPITA, AUGMENTED MODEL

$$\ln \left[\frac{Y(t)}{L(t)} \right] = \ln A(0) + gt - \frac{\alpha + \beta}{(1 - \alpha - \beta)} \ln(n + g + \delta) + \frac{\alpha}{(1 - \alpha - \beta)} \ln(s_k) + \frac{\beta}{(1 - \alpha - \beta)} \ln(s_h)$$

Equation 5 is the basis for the empirical specification of the Augmented Solow Model, which is given by Equation 6.

EQUATION 6
EMPIRICAL ESTIMATING EQUATION, AUGMENTED MODEL

$$\ln \left(\frac{Y(t)}{L(t)} \right)_i = \gamma_0 + \gamma_1 \ln(\bar{s}_{k_i}) + \gamma_2 \ln(\bar{n}_i + g + \delta) + \gamma_3 \ln(\bar{s}_{h_i}) + \varepsilon_i$$

Here, \bar{s}_h is the average rate of human capital accumulation (or level of human capital) for country i . The expected results are $\gamma_1 > 0$, $\gamma_2 < 0$, and $\gamma_3 > 0$. If the level of human capital, rather than the accumulation rate of human capital, is used the theoretical predictions of the magnitude of the coefficients on all variables in the model differ, while the predicted signs remain the same. If the level of human capital is used, Equation 5 changes slightly. See MRW (p. 417-418) for a full discussion.

In order to test whether the linear specification is appropriate, a nonlinear model is specified and GAM is used for estimation. Originally proposed by Hastie and Tibshirani (1990), a generalized form of GAM is given in Equation 7, where $f_j(x_j, m)$ is a polynomial smoothing function degree m fit to the x_j series. If $m = 1$ then $f_j(x_j, 1) \equiv x_j$, but if $m > 1$ then the x variable is smoothed. Once the $f_j(x_j, m)$ vectors for $j = 1, k$ are calculated, OLS is used to estimate γ_j , $j = 0, k$. The GAM algorithm also allows for significance tests of the estimated coefficients using asymptotic standard errors corrected for degrees of freedom. See Hastie and Tibshirani (1990, p. 127) for a full discussion.

EQUATION 7
THE GENERALIZED ADDITIVE MODEL

$$E(y) = g(x_1, x_2, \dots, x_k) = \gamma_0 + \gamma_1 f_1(x_1, m) + \gamma_2 f_2(x_2, m) + \dots + \gamma_k f_k(x_k, m) + e$$

A key feature of the GAM algorithm is that the gain from removing the linearity assumption of one variable, j , in an otherwise linear model can be tested. This is accomplished by calculating $e'e$ when $m_j > 1$ and comparing this to $e'e$ when $m = 1$. A statistical test is used to determine whether the increase in $e'e$ when $m = 1$, holding the functional form of all other variables fixed, is statistically significant.

Stokes (2008) suggests that GAM should be used as a routine diagnostic procedure. Stokes further suggests that it is theoretically sound to use the GAM coefficients to estimate out of sample. A critical question is whether a nonlinear model estimated with GAM provides better out of sample forecasts than a linear model estimated with OLS. If the relationship between the variables is truly nonlinear, and if GAM has accurately estimated the functional form of the nonlinear relationship, then the model estimated with

GAM should provide better out of sample predictions. On the other hand, if GAM has over-fit the data, the out of sample predictions from the nonlinear model will be poor.

The nonlinear estimating equations are variants of Equations 3 and 6, where the right-hand side variables are smoothed as described above. The nonlinear equation for the Textbook Solow Model is given by Equation 8, where f_i is a smoothing function on the i^{th} variable. Similarly, the nonlinear equation for the Augmented Solow Model is given by Equation 9. The basic GAM results reported in subsequent sections are insensitive to the choice of the degree of smoothing, so degree three has been used throughout.

EQUATION 8
GAM ESTIMATING EQUATION, TEXTBOOK MODEL

$$\ln\left(\frac{Y(t)}{L(t)}\right)_i = \gamma_0 + \gamma_1 f_1(\ln(\bar{s}_i), m) + \gamma_2 f_2(\ln(\bar{n}_i + g + \delta), m) + \varepsilon_i$$

EQUATION 9
GAM ESTIMATING EQUATION, AUGMENTED MODEL

$$\ln\left(\frac{Y(t)}{L(t)}\right)_i = \gamma_0 + \gamma_1 f_1(\ln(\bar{s}_{k_i}), m) + \gamma_2 f_2(\ln(\bar{n}_i + g + \delta), m) + \gamma_3 f_3(\ln(\bar{s}_{h_i}), m) + \varepsilon_i$$

The following approach is used to generate the out-of-sample forecasts. For forecasting using the linear specification, Equations 3 and 6 are estimated with OLS using data for the period 1960 to 1985. The coefficients are then saved and applied to the 1960-2000 data, as in Equation 10, where $\hat{y}_{OLS_forecast}$ is a vector of forecasted log incomes for the 1960-2000 period, $X_{1960-2000}$ is the matrix of explanatory variables for the 1960-2000 period, and $\Gamma_{1960-1985}^{OLS}$ is the vector of coefficients from the linear model of the 1960-1985 period.

EQUATION 10
OLS OUT OF SAMPLE FORECASTING

$$\hat{y}_{OLS_forecast} = X_{1960-2000} \Gamma_{1960-1985}^{OLS}$$

Similarly, nonlinear forecasts are obtained by first generating coefficients and smoothing functions by estimating (with GAM) Equations 8 and 9 with the 1960-1985 data. Then, using updated data, the estimated smoothing functions are applied to the explanatory variables, and the smoothed vectors are multiplied by the 1960-1985 coefficients, as in Equation 11, where $[sX]_{1960-2000}$ is the matrix of smoothed explanatory variables obtained by applying the 1960-1985 smoothing functions to the 1960-2000 data, and $\Gamma_{1960-1985}^{GAM}$ is the vector of coefficients from the nonlinear model.

EQUATION 11
GAM OUT OF SAMPLE FORECASTING

$$\hat{y}_{GAM_forecast} = [sX]_{1960-2000} \Gamma_{1960-1985}^{GAM}$$

DATA

For comparison of the results over different samples and time periods, five distinct cross-section data sets are defined and named *Data Set A* through *Data Set E*. These data sets are described in detail below and summarized in Table 1.

MRW use a cross-section of 98 non-oil producing countries for the period 1960-1985. The MRW data are from an early version of the Penn World Table (PWT) constructed by Summers and Heston (1988). MRW measure n as the average rate of growth of the working age (15-64) population, s as the average share of real investment in real GDP, and Y/L as real GDP in 1985 divided by the working age population in that year. MRW measure human capital accumulation using a proxy that measures the approximate percentage of the working age population that is in secondary school. *Data Set A* is the MRW data as published in the 1992 paper.

In order to update the analysis to include years after 1985, a data set for the period 1960-2003 is constructed primarily from the PWT, Version 6.2, documented in Summers, et. al. (2006). PWT 6.2 is the source of data on real GDP per capita, investment share of GDP, and population size. Data on the percentage of the total population that is of working age was obtained from the World Bank World Development Indicators (2008). *Data Set B* contains 97 countries for which all data series are available for the 1960-2003 period.

Two measures of human capital accumulation augment the PWT 6.2 data. First, making an assumption that the approximate rate of human capital accumulation has not changed significantly since 1985, the MRW human capital accumulation measure is transplanted into the updated data set. This augmented data set, *Data Set C*, contains 83 countries which are contained in both the MRW data and the updated PWT 6.2 data. For comparability, *Data Set D* is a subset of the original MRW data (*Data Set A*), but contains only the 83 countries which are also contained in the PWT 6.2 data. The second measure of human capital uses data on the average years of education for the population over 25, averaged over the study period, from Barro and Lee (2001). This second augmented data set, *Data Set E*, contains 75 countries; these are countries contained in the 97 country PWT 6.2 data set for which the Barro-Lee data is also available.

Two measures of human capital accumulation are used because measures of this variable are thought to be flawed. Although the Barro-Lee data is acknowledged to be an improvement upon MRW's rough measure, it still is imperfect. Measuring human capital in two ways provides a robustness check on both the linear and the nonlinear results for this variable. It is important to note, however, that the MRW measurement corresponds to the rate of accumulation of human capital, while the Barro-Lee data corresponds to the *level* of human capital. While this does not change the basic expected results of the empirical equations, it does change the expected magnitude of the coefficients. See Mankiw, et. al. (1992, p. 418) for a discussion. The data sets used for estimation are briefly described in Table 1.

RESULTS OF ESTIMATING THE LINEAR AND NONLINEAR MODELS

To establish a baseline of comparability between the data sets under study, estimation of the linear specification of both the Textbook and the Augmented Solow Models for the period 1960-1985 is replicated. Table 2 gives the results of the OLS estimation of Equation 3, the empirical specification of the Textbook Solow Model, using various data sets. Columns 1 and 2 give the original MRW results alongside the results when *Data Set D*, which contains the 83 countries that overlap the PWT 6.2 data, is used. Reducing the sample size from 98 to 83 countries does not make a substantive difference in the OLS results, as the coefficients are all statistically equal. The explanatory power of the model improves slightly ($\bar{R}^2 = 0.59$ vs. 0.64), but this is the only notable change.

Columns 3 and 4 of Table 2 report the OLS results of Equation 3 using data from the updated version (6.2) of the Penn World Table for the period 1960-1985. A comparison of Columns 1 and 3 shows that

TABLE 1
DESCRIPTION OF DATA SETS A THROUGH E

<i>Data Set</i>	<i>Description</i>	<i>N</i>
<i>A</i>	Full sample from Mankiw, Romer, and Weil (1992); Period 1960-1985	98
<i>B</i>	Full sample from Penn World Table, v. 6.2; Period 1960-2003	97
<i>C</i>	Reduced sample from Penn World Table, v. 6.2, containing countries from Data Sets A and B that overlap. Augmented with the variable "SCHOOL" from Mankiw, Romer, and Weil. Period 1960-2003	83
<i>D</i>	Reduced sample from Mankiw, Romer, and Weil containing countries from Data Sets A and B that overlap; Period 1960-1985	83
<i>E</i>	Reduced sample from Penn World Table, v. 6.2, containing countries for which Barro-Lee (2001) data on educational attainment also is available. Period 1960-2000	75

when the OLS model is estimated with the full samples, there is a significant difference in the estimated coefficient of the savings rate, although the period under study is the same. The original MRW estimate of 1.42 is statistically different than the coefficient of 0.88 obtained using the updated data. A possible explanation for this difference is the non-comparability of the samples. Since Data Sets A and B contain only 83 common countries, different coefficients are not unexpected when the full (98 or 97 country) samples are used. However, an examination of Columns 2 and 4 which estimate the OLS model for only the 83 overlapping countries shows that the statistical difference in the coefficients on savings persists even when the samples are identical. The explanation for this difference appears to be a sizable difference in the average savings rate obtained from two different versions of the Penn World Table.

A similar result arises from the OLS estimation of the Augmented Solow Model, Equation 6. Table 3 gives these results. Again, comparing Columns 1 and 2 demonstrates that dropping the non-overlapping countries from the original MRW sample makes little difference in the estimated coefficients. However, there is a significant difference between the estimated coefficients on the log of savings between Data Sets C (the data from the updated version of the Penn World Table) and D (the comparable sample from the original MRW paper) even when looking at an identical period (1960-1985). Comparison of Columns 2 and 3 shows this difference. Coefficients on the population growth rate and the school variable are statistically equal. The coefficients reported in Column 6 are somewhat different from those in Columns 2 and 3, but this difference is expected as Data Set E contains levels rather than growth rates of human capital.

Tables 2 and 3 also contain OLS results when the Textbook and Augmented Solow Models are estimated over updated periods. The OLS results for the period 1960-2000 are similar in nature to the results from the shorter 1960-1985 period, with the coefficient on log savings positive and significant and the coefficient on log population growth negative and significant. This demonstrates that the basic

TABLE 2
OLS ESTIMATION OF THE TEXTBOOK SOLOW MODEL

Dependent Variable: Log GPD per working-age person in terminal year of period								
Data	MRW		PWT62		PWT62		PWT62	
Period	1960-1985		1960-1985		1960-2000		1960-2003	
<i>Data Set</i>	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<i>Data Set</i>	<i>A</i>	<i>D</i>	<i>B</i>	<i>C</i>	<i>B</i>	<i>C</i>	<i>B</i>	<i>C</i>
Constant	5.43 (1.59) ^a	6.50 (1.60)	4.98 (1.55)	5.58 (1.57)	2.59 (1.61)	3.43 (1.58)	1.97 (1.65)	2.79 (1.60)
ln(I/GDP)	1.42 ^b (0.14)	1.52 ^b (0.15)	0.88 ^b (0.11)	0.95 ^b (0.11)	0.97 ^b (0.13)	1.08 ^b (0.12)	0.95 ^b (0.13)	1.05 ^b (0.13)
ln($n+g+\delta$)	-1.99 ^b (0.56)	-1.68 ^b (0.56)	-2.16 ^b (0.56)	-1.99 ^b (0.57)	-3.20 ^b (0.57)	-2.97 ^b (0.56)	-3.42 ^b (0.58)	-3.19 ^b (0.56)
\bar{R}^2	0.59	0.64	0.49	0.56	0.59	0.66	0.59	0.67
Countries	98	83	97	83	97	83	97	83

^a Standard errors in parentheses.

^b Significant at the 99% level.

predictions of the Solow model hold for the longer period. These results replicate findings of other authors (Bernanke and Gurkaynak 2001; Karras 2008). Notice that results for the period 1960-2003 (the longest period for which data is available) are nearly identical to results for the period 1960-2000. This can be seen in the comparison of Columns 5 and 6 with 7 and 8 in Table 2, and also in the comparison of Column 4 with Column 5 in Table 3. For this reason, the nonlinear estimation discussed next is limited to the 1960-2000 period; there is no expectation that adding the additional three years of data would change the basic findings.

In summary, with the exception of the estimated coefficient on the log of savings variable, the OLS results originally presented by MRW hold across different versions of the 1960-1985 data. The results also hold over the longer (1960-2003) period. These findings increase confidence in the suitability of the simple linear model; if the model is truly an accurate description of the determinants of steady state income, then it is expected that the basic results would not be sensitive to the time period under study.

The question of interest in this paper, however, is not only whether the original results hold for updated data, but also whether the implicit assumptions of the linear model have an important effect on the empirical results. Use of GAM allows the relaxation of the linearity assumption. Rather than imposing the restriction that the coefficient on each variable is constant across the entire range of that variable, GAM allows the coefficients to vary by applying a data-determined smoothing function to each explanatory variable before estimation. Both the Textbook and the Augmented Solow Models are estimated using GAM, starting with the period 1960-1985, using the original MRW data as well as the updated version of the data (for the same period) from PWT 6.2.

Results of the GAM estimation of the Textbook Solow Model are reported in Table 4. GAM coefficients are reported and can be interpreted as the coefficients on the smoothed explanatory variables. Asymptotic standard errors (corrected for degrees of freedom) are reported in parentheses below the

TABLE 3
OLS ESTIMATION OF THE AUGMENTED SOLOW MODEL

Dependent Variable: Log GPD per working-age person in terminal year of period							
Data	MRW		PWT62, augmented with MRW's SCHOOL variable			PWT62, augmented with Barro-Lee data	
Period:	1960-1985		1960-1985	1960-2000	1960-2003	1960-1985	1960-2000
<i>Data Set</i>	(1) <i>A</i>	(2) <i>D</i>	(3) <i>C</i>	(4) <i>C</i>	(5) <i>C</i>	(6) <i>E</i>	(7) <i>E</i>
Constant	6.84 (1.18) ^a	6.91 (1.26)	6.30 (1.23)	4.14 (1.25)	3.45 (1.29)	5.54 (1.20)	3.70 (1.25)
ln(I/GDP)	0.70 ^b (0.13)	0.78 ^b (0.16)	0.30 ^c (0.13)	0.36 ^c (0.14)	0.36 ^c (0.14)	0.27 ^c (0.12)	0.30 ^c (0.14)
ln($n+g+\delta$)	-1.75 ^b (0.42)	-1.71 ^b (0.44)	-2.00 ^b (0.44)	-2.93 ^b (0.44)	-3.16 ^b (0.45)	-1.21 ^b (0.45)	-1.88 ^b (0.46)
ln(SCHOOL)	0.65 ^b (0.07)	0.58 ^b (0.08)	0.66 ^b (0.09)	0.64 ^b (0.09)	0.62 ^b (0.09)	0.73 ^b (0.10)	0.86 ^b (0.12)
R ²	0.78	0.78	0.73	0.79	0.79	0.71	0.75
Countries	98	83	83	83	83	75	75

^a Standard errors in parentheses.

^b Significant at the 99% level.

^c Significant at the 95% level.

coefficients. See Hastie and Tibshirani (1990, p. 127) for overview of distribution and inference when using GAM. The sum of squared residuals reported with each explanatory variable in the body of Table 4 is the sum of squared residuals of the model if the current variable is constrained to be linear while all other variables in the model are smoothed. The row labeled 'sig' reports a p-value for a significance test of whether the sum of squared residuals when the variable is forced to enter the model linearly is significantly higher than the sum of squared residuals when all variables are smoothed. This is a statistical test, where the null hypothesis is that the sum of squared residuals is equal regardless of whether the variable is constrained to linearity. The alternative hypothesis is that the sum of squared residuals increases when the variable is constrained to linearity. The p-value indicates the confidence level with which the null hypothesis can be rejected.

The sum of squared residuals when all variables are smoothed is reported at the bottom of Table 4 as "GAM e'e." This value can be qualitatively compared to the OLS sum of squared residuals. For a statistical comparison, however, the significance tests should be consulted as these have been corrected for the loss of degrees of freedom implicit in the GAM estimation. A high p-value for the significance test suggests that imposing the linearity assumption on the current variable has a significant effect on the sum of squared residuals of the estimated model. Similarly, a low p-value indicates that there is no gain from estimating the variable non-linearly.

TABLE 4
GAM ESTIMATION OF THE TEXTBOOK SOLOW MODEL

Dependent Variable: Log GPD per working-age person in terminal year of period						
Period	1960-1985		1960-1985		1960-2000	
<i>Data Set</i>	(1) <i>A</i>	(2) <i>D</i>	(3) <i>B</i>	(4) <i>C</i>	(5) <i>B</i>	(6) <i>C</i>
Constant	5.31 (1.52) ^a	6.46 (1.53)	5.22 (1.49)	5.82 (1.47)	2.72 (1.61)	3.64 (1.57)
ln(I/GPD)	1.35 (0.14)	1.47 (0.14)	0.84 (0.11)	0.90 (0.11)	0.95 (0.13)	1.05 (0.12)
e'e	40.62	29.31	48.01	34.14	48.37	34.03
sig	0.51	0.44	0.78	0.95	0.26	0.58
ln($n+g+\delta$)	-1.98 (0.54)	-1.67 (0.54)	-2.03 (0.54)	-1.86 (0.53)	-3.14 (0.57)	-2.87 (0.55)
e'e	44.33	32.37	49.37	33.85	49.21	33.94
sig	0.99	0.98	0.93	0.93	0.59	0.55
R^2	0.65	0.70	0.56	0.64	0.61	0.69
Countries	98	83	97	83	97	83
GAM e'e	39.57	28.54	45.75	30.93	47.70	32.80
OLS e'e	45.11	33.01	51.53	36.93	49.81	35.10

^a Asymptotic standard errors (corrected for degrees of freedom) in parentheses.

For example, in Column 1, for the variable $\ln(I/GDP)$ (the log of average savings), the reported sum of squared residuals is 40.62. The interpretation is that if this variable is constrained to enter the model linearly (with no smoothing), the sum of squared residuals of the model would jump from 39.57 (the GAM e'e) to 40.62. The p-value ('sig') of 0.51 suggests that this increase in the sum of squares is not statistically significant. By contrast, for the variable $\ln(n+g+\delta)$ (the log of population growth), the reported e'e is 44.33. If this variable was constrained to enter the model linearly, the sum of squared residuals would increase from 39.57 to 44.33. The p-value of 0.99 indicates that this difference is statistically significant at the 0.01 level.

Starting with the period 1960-1985, Columns 1 and 2 of Table 4 report the results of GAM testing on the original MRW data, both the full 98-country sample and the reduced 83 country sample. The savings variable is not significantly non-linear, as there is no significant increase in the sum of squared residuals if linearity is imposed. The results of the significance testing are 0.51 for the 98 country sample and 0.44 for the 83 country sample. However, the population growth variable *is* significantly non-linear. The results of the significance testing on this variable are 0.99 for the 98 country sample and 0.98 for the 83 country sample. This suggests that imposing a constant coefficient on the population growth variable is not appropriate and that the linear specification is likewise not appropriate.

Columns 3 and 4 of Table 4 show the GAM results for the 1960-1985 period using data from version 6.2 of the Penn World Table. The expectation is that these results should be very similar to the results reported in Columns 1 and 2; however, the results are surprisingly different. Of particular interest is a

comparison of Columns 2 and 4, where the countries included in the sample are identical. Using Data Set D, savings is linear while population is non-linear. Using Data Set C, savings is significantly nonlinear while population growth is only marginally significantly nonlinear. Just as in the OLS results, these differences can best be explained by differences in the measurement of average savings between the two versions of the Penn World Tables.

To provide some insight into whether these differences are substantive or merely statistical, an examination of leverage plots showing the effect of the explanatory variables on income is useful. The slope of the leverage plot gives the effect on income at a given level of the explanatory variable. Figure 1 shows the effect of log savings and log population growth on log income using Data Set A. Figure 2 shows the same effects using Data Set B. Figure 3 shows the effects using Data Set C, and Figure 4 shows the effects using Data Set D. Notice that although there are statistical differences in the GAM results (as reported in Table 4), the graphical analysis indicates the results are qualitatively similar across the various data sets. The effect of log savings is consistently positive, while the effect of population growth is negative to a point, and positive beyond that point.

Table 5 gives the results of the GAM estimation of the Augmented Solow Model. Columns 1 and 2 give results for Data Sets A and D, respectively. Notice that GAM estimation using Data Set A suggests that both population growth and schooling are significantly non-linear, while savings is linear. (P-values

FIGURE 1
TEXTBOOK SOLOW MODEL, DATA SET A, PERIOD 1960-1985

Effect of Variables on Log Income

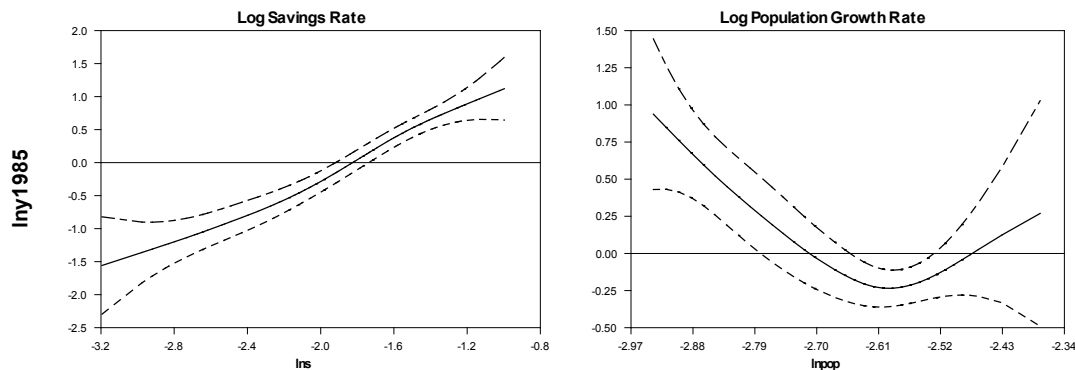
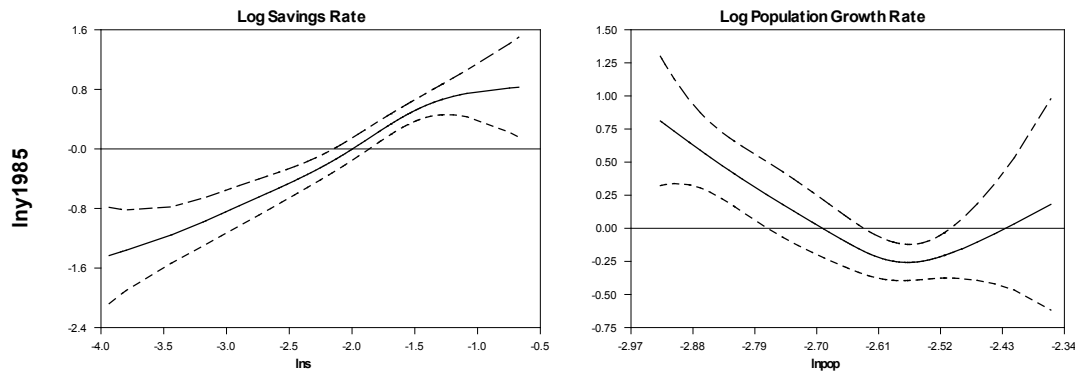


FIGURE 2
TEXTBOOK SOLOW MODEL, DATA SET B, PERIOD 1960-1985

Effect of Variables on Log Income



solid line shows effect, dashed lines show confidence intervals

FIGURE 3
TEXTBOOK SOLOW MODEL, DATA SET C, PERIOD 1960-1985
Effect of Variables on Log Income

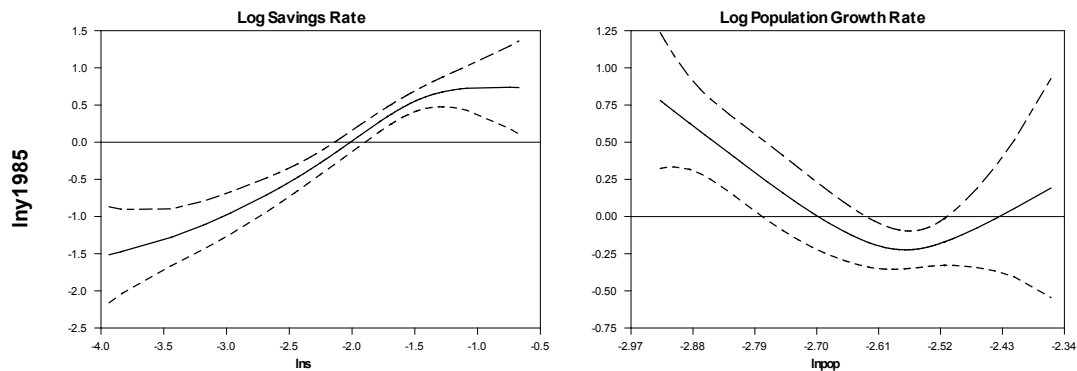
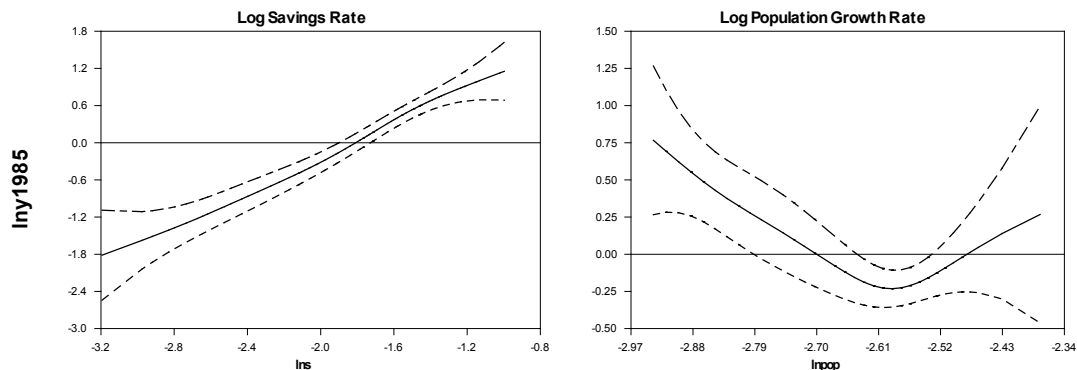


FIGURE 4
TEXTBOOK SOLOW MODEL, DATA SET D, PERIOD 1960-1985
Effect of Variables on Log Income



solid line shows effect, dashed lines show confidence intervals

are 0.99, 0.98, and 0.72 respectively.) In this case, dropping the 15 countries and re-estimating with Data Set D makes a difference in the results. Savings is still linear with p-value 0.66, population growth is still nonlinear with p-value 0.99, but now schooling appears linear, with a p-value of 0.83.

Columns 2, 3, and 5 of Table 5 provide a comparison of GAM estimation for three different data sets for the 1960-1985 period using Data Sets D, C, and E respectively. In all three cases, savings is not significantly nonlinear. Data Sets D and E agree that population growth is significantly nonlinear, while Data Sets C and E agree with Data Set A that human capital accumulation is significantly nonlinear. Although there are some statistical differences across the different data sets, the general result is that log savings affects log income linearly, while log population growth and log human capital accumulation affect log income nonlinearly. Figures 5, 6, 7, and 8 show the leverage plots for the three explanatory variables for Data Sets A, D, C, and E. The leverage plots support the general finding that the behavior of the nonlinearity is consistent across the data sets.

A difference in the GAM results is observed for the 1960-2000 period. In Table 4, Columns 5 and 6 report the results of GAM estimation of the Textbook Solow Model for Data Sets B and C. The results are qualitatively similar for the two data sets; dropping the 14 countries not included in the MRW data does not make a difference in the results of GAM estimation. Importantly, for the 1960-2000 period, neither the log savings nor the log population growth variable is significantly nonlinear. This suggests that the

TABLE 5
GAM ESTIMATION OF THE AUGMENTED SOLOW MODEL

Dependent Variable: Log GPD per working-age person in terminal year of period						
Data	MRW		PWT62, augmented with MRW's SCHOOL Variable		PWT62, augmented with Barro-Lee data	
Period	1960-1985		1960-1985	1960-2000	1960-1985	1960-2000
<i>Data Set</i>	(1) <i>A</i>	(2) <i>D</i>	(3) <i>C</i>	(4) <i>C</i>	(5) <i>E</i>	(6) <i>E</i>
Constant	7.44 (1.09)	7.66 (1.18)	6.97 (1.14)	5.29 (1.21)	7.18 (1.08)	5.40 (1.21)
ln(I/GDP)	0.63 (0.12) ^a	0.72 (0.15)	0.23 (0.12)	0.30 (0.14)	0.23 (0.11)	0.28 (0.14)
e'e sig	20.63 0.72	16.92 0.66	19.00 0.83	19.42 0.67	14.74 0.80	16.83 0.47
ln($n+g+\delta$)	-1.49 (0.38)	-1.39 (0.41)	-1.72 (0.41)	-2.52 (0.42)	-0.55 (0.41)	-1.18 (0.45)
e'e sig	22.05 0.99	18.53 0.99	19.26 0.89	18.63 0.05	15.69 0.97	16.38 0.07
ln(SCHOOL)	0.66 (0.07)	0.59 (0.08)	0.67 (0.09)	0.71 (0.09)	0.76 (0.09)	0.94 (0.15)
e'e sig	21.82 0.98	17.28 0.83	19.66 0.95	20.42 0.94	16.19 0.99	18.47 0.97
R^2	0.82	0.83	0.79	0.83	0.79	0.79
Countries	98	83	83	83	75	75
GAM e'e	20.07	16.17	17.77	18.54	13.76	16.27
OLS e'e	24.23	20.12	22.46	21.29	18.52	18.85

^a Asymptotic standard errors (corrected for degrees of freedom) in parentheses.

OLS specification is appropriate for the Textbook Solow Model for the 1960-2000 period, which is a contradiction of the findings for the 1960-1985 period, where the effect of population growth rate was shown to be nonlinear. Figures 9 and 10 show the leverage plots for the 1960-2000 period, where the effects of the explanatory variables are constant across the entire range of the variables.

The results of GAM estimation of the Augmented Solow Model for the period 1960-2000 are similar in nature. Columns 4 and 6 of Table 5 give GAM results for this period, with human capital accumulation measured in two ways, first as a growth rate (Column 4) and second as a level (Column 6). The results are qualitatively similar: both log savings and log population growth are not significantly non-linear, but log human capital *is* significantly non-linear. The fact that these results agree with the results of the GAM estimation of the Textbook Model together with the consistency of the results across different samples (83

FIGURE 5
AUGMENTED SOLOW MODEL, DATA SET A, PERIOD 1960-1985
Effect of Variables on Log Income

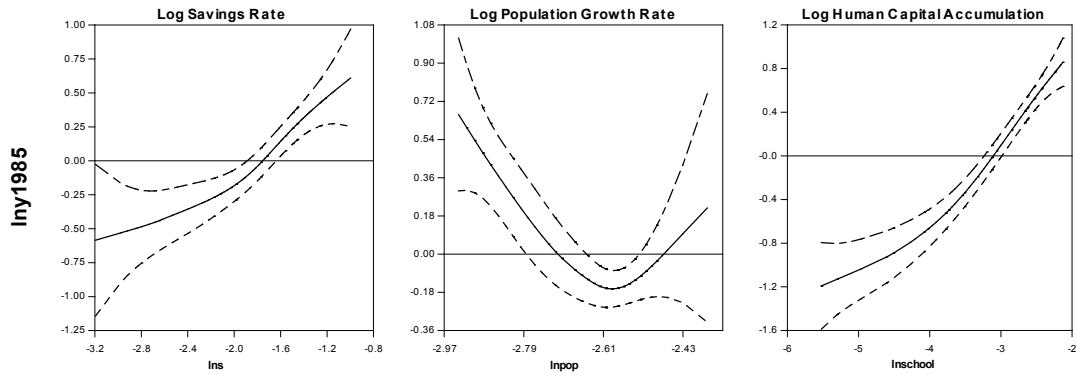


FIGURE 6
AUGMENTED SOLOW MODEL, DATA SET D, PERIOD 1960-1985
Effect of Variables on Log Income

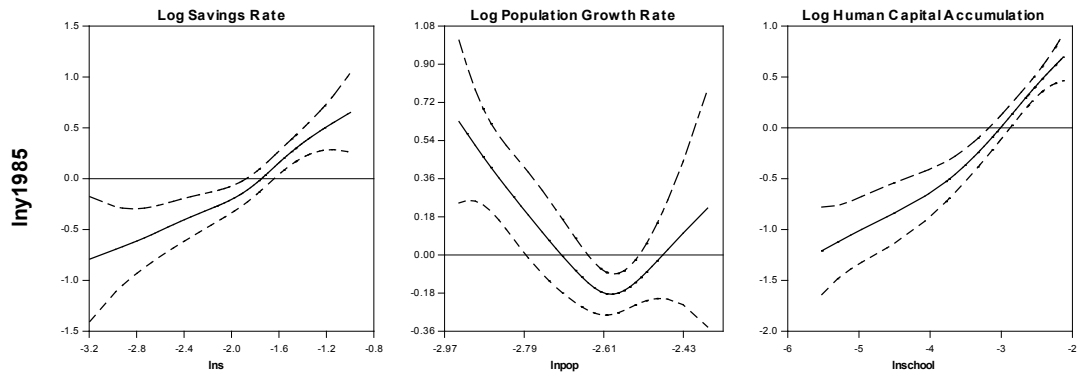
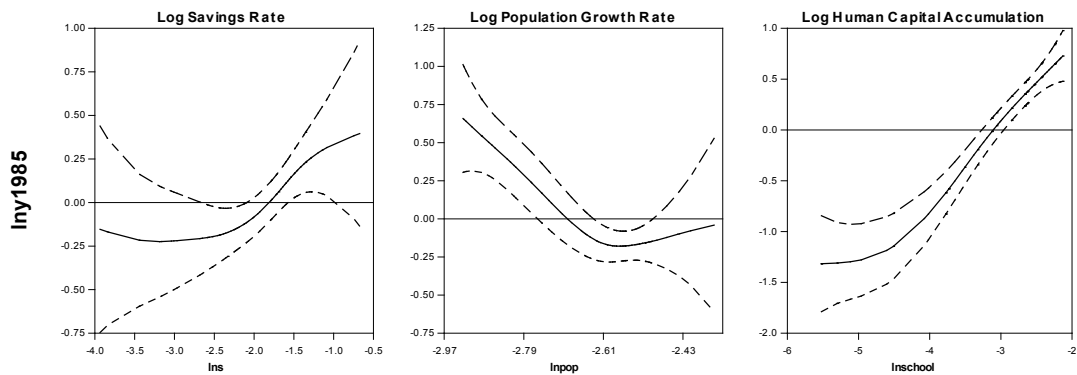


FIGURE 7
AUGMENTED SOLOW MODEL, DATA SET C, PERIOD 1960-1985
Effect of Variables on Log Income



solid line shows effect, dashed lines show confidence intervals

FIGURE 8
AUGMENTED SOLOW MODEL, DATA SET E, PERIOD 1960-1985

Effect of Variables on Log Income

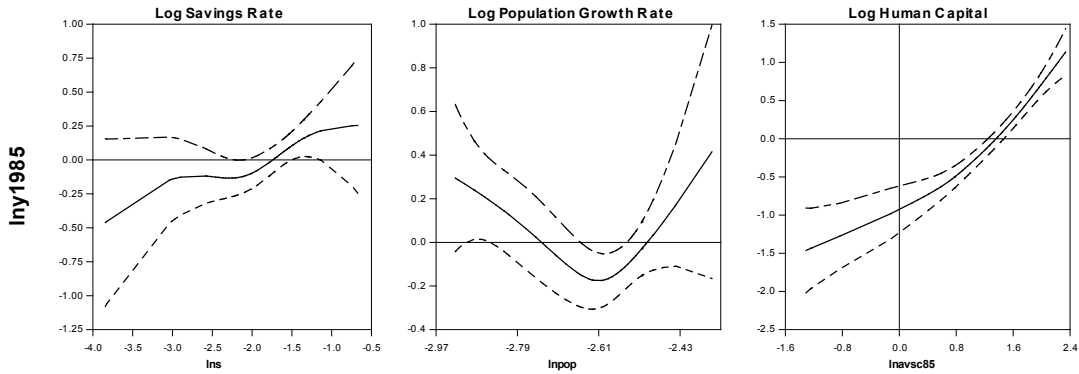


FIGURE 9
TEXTBOOK SOLOW MODEL, DATA SET B, PERIOD 1960-2000

Effect of Variables on Log Income

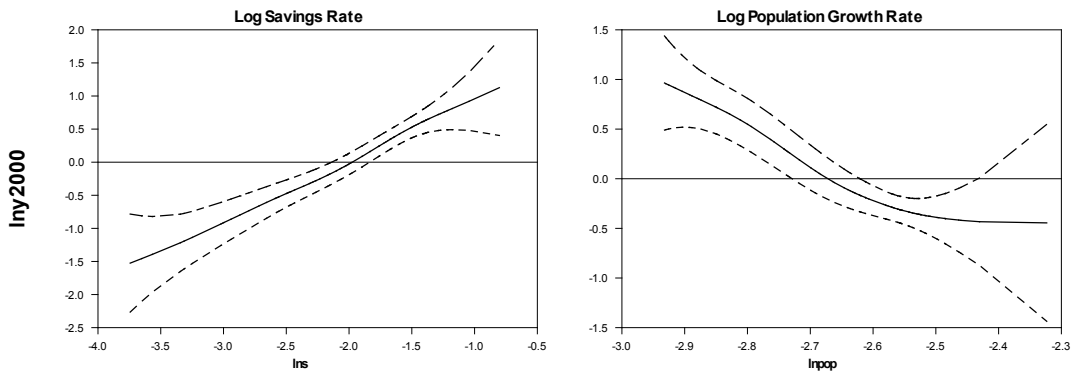
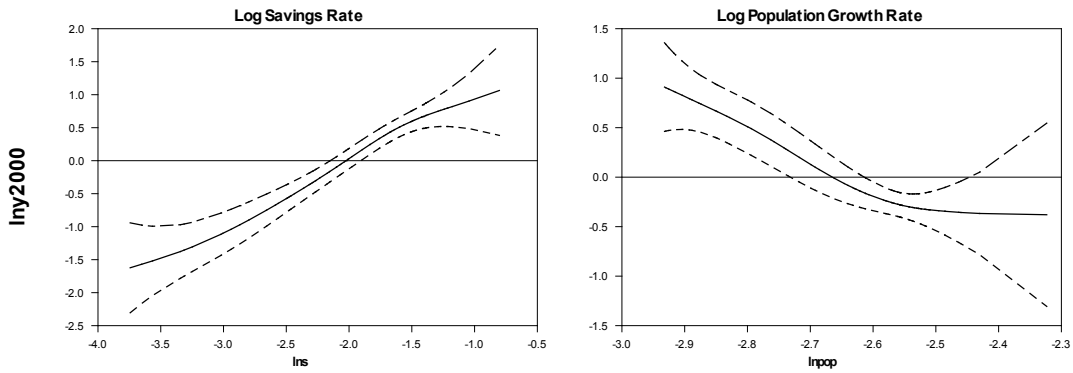


FIGURE 10
TEXTBOOK SOLOW MODEL, DATA SET C, PERIOD 1960-2000

Effect of Variables on Log Income



solid line shows effect, dashed lines show confidence intervals

FIGURE 11
AUGMENTED SOLOW MODEL, DATA SET C, PERIOD 1960-2000
Effect of Variables on Log Income

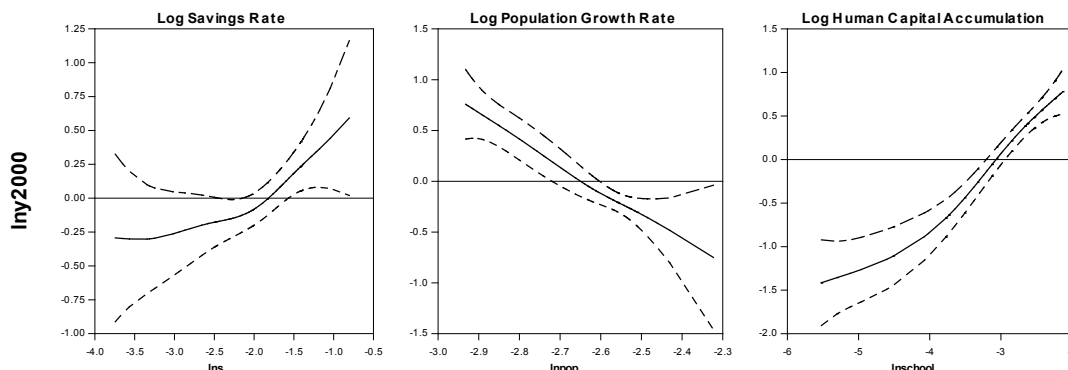
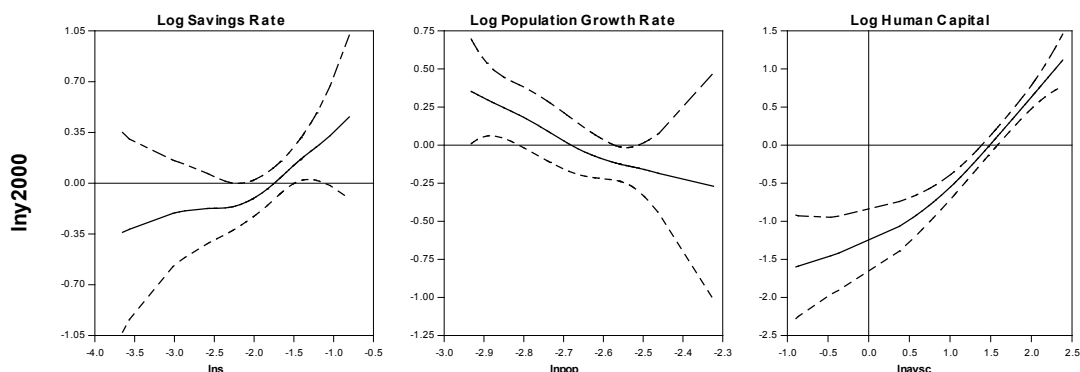


FIGURE 12
AUGMENTED SOLOW MODEL, DATA SET E, PERIOD 1960-1985
Effect of Variables on Log Income



solid line shows effect, dashed lines show confidence intervals

vs. 75 countries) and measurement strategies for human capital lend strong support to the argument that this result is an accurate description of the relationship of human capital and steady state income.

Figures 11 and 12 show the leverage plots for the Augmented Solow Model in the 1960-2000 period. As in Figures 9 and 10, the effect of log savings is represented by a constantly sloped upward sloping line and the effect of log population growth is represented by a constantly sloped downward sloping line. Of interest here is the shape of the leverage plot for human capital. The effect of human capital on income is positive and increasing with the level of human capital. This suggests that while the effect of human capital accumulation on income is always positive, this effect is more pronounced in economies where there is already a substantial investment rate in and/or stock of human capital.

RESULTS OF OUT-OF-SAMPLE FORECASTING

Following Temple (2000), the empirical models developed for the 1960-1985 period can be applied to the 1960-2000 data to test whether the linear or the nonlinear model performs better. Because the use of a nonlinear model leads to a loss of simplicity, and because the results are more difficult to interpret than linear results, it is important to consider whether the nonlinear model provides a substantive improvement over the linear, or whether the gain is mainly statistical. Another concern is the possibility of over-fitting,

since the nonlinear model is developed based on the input data set. One way of addressing these questions is to study whether the model estimated with GAM improves the out-of-sample predictive power of the model of steady state income relative to the model estimated with OLS. If the nonlinear model is over-fit, then the linear model should perform better out-of-sample since the nonlinear model will be particular to the data set that was used to generate it. If the nonlinear model predicts more accurately than the linear model, this would suggest that GAM has uncovered true nonlinear relationships between the variables in the model, and that these relationships persist when additional years of data are included.

It is a simple forecasting exercise to estimate linear and nonlinear models (with OLS and GAM, respectively) from Data Set D, the 83 country sub-set of the MRW data set, for both the Textbook and the Augmented Solow Models. Coefficients are reported in Tables 2 – 5. To test the predictive power of the linear versus the nonlinear model, these coefficients as well as the GAM smoothing functions are applied to Data Set C, the corresponding 83 country sample from Version 6.2 of the Penn World Table for the 1960-2000 period. Equations 10 and 11 in Section 2 describe the procedure used to generate the forecasts.

For the Textbook Solow Model using these two data sets, the nonlinear model outperforms the linear model by about 10% when used to forecast future income based on out-of-sample data. The sum of squared error for the out-of-sample predictions using OLS is 166.34, while the sum of squared error using GAM 148.58. For the Augmented Solow Model the results are similar. The sum of squared error for the out of sample predictions using OLS is 123.75, while the sum of squared error using GAM 110.40. Again, GAM provides an improvement of about 10% in out-of-sample forecasting. These results suggest that the nonlinear model based on the 1960-1985 period is not over-fit but rather has captured true relationships between income and the independent variables (savings, population growth, and education). If a researcher or policy maker was interested in predicting future income based on a current model, this result suggests that the researcher could expect a more accurate forecast using a nonlinear model versus an linear model.

CONCLUSIONS

Results of nonlinear estimation using GAM change depending on what time period is studied. Although GAM estimation for the period 1960-1985 suggests that the population growth rate affects steady state income in a nonlinear way, this result disappears for the 1960-2000 period. By contrast, the effect of human capital is not constant across the levels of human capital, regardless of the time period under study. The effect of increasing human capital accelerates as the level of the human capital variable increases. This result is found across multiple data sets representing both different time periods and different measurement strategies for human capital. The result is also in keeping with an intuitive notion that while the effect of human capital accumulation will always be positive, the marginal effect of education will be stronger as the existing pool of human capital grows.

In addition, a nonlinear model estimated with GAM performs about 10% better than a linear model in out of sample predictions for both the Textbook and the Augmented Solow Models. This suggests that nonlinear modeling and GAM estimation are useful tools for forecasting. It also demonstrates that the models developed using the 1960-1985 data are relevant outside of this period, suggesting the GAM is uncovering true relationships in between the variables as opposed to over-fitting the initial sample.

There are several questions raised here that could be researched further. Of particular interest is the development of a theoretical explanation for the changing coefficients suggested by GAM.

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