Exploiting Long Term Price Dependencies for Trading Strategies

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The following research uses properties of cointegrated time series that serve as a basis for a daily trading strategy. Its practical implementation is illustrated using the daily closing prices of four world stock market indices. In-sample (as well as out-of-sample) results show that the long-term dependencies of financial time series can be profitably exploited in a variant of "pairs trading" strategy. This paper includes an extended empirical study that shows the strategy's performance as a function of its parameters. The backtests presented show the daily profit-and-loss results for the period 2001 to 2006. During that time, the strategy significantly outperformed a simple buy-and-hold of the individual indices.

INTRODUCTION

Hedge funds have become a popular alternative investment, partly because they use a variety of strategies that exploit anomalies in the financial markets. Focus has shifted to a group of strategies that rely on "quantitative analysis" (also known as "statistical arbitrage"), which uses statistical and optimization techniques to discover and construct new algorithms. These algorithms take advantage of the short-term deviation from "fair" securities' prices. *Pairs trading*, one such quantitative strategy, identifies securities that generally move together but are currently diverging. Such strategies infer from the existing long-term relationship among the securities, that they will eventually reconverge.

Many equity hedge funds are using or have used approaches that rely on this convergence tendency. One common method for finding such spreads is based on correlation. Since correlation is a short-term measure, frequent rebalancing of a portfolio is necessary to reflect the constantly changing spreads, which tends to incur high transaction costs. Additionally, the correlation could be a very unreliable measure during financial market crises, like the collapse of Long Term Capital Management in 1998.

Cointegration is a measure of the long-term dependencies in financial time series. Since the seminal work of Engle & Granger (1987, 1991), many researchers in finance and economics have used cointegration to model the dependencies between securities prices. Several statistical test procedures for cointegration have been developed (see Engle & Granger 1991). Tsay (2005) presented numerous examples and a detailed justification of using cointegration in modeling financial time series. Alexander et al. (2002) used cointegration to construct an index tracking portfolio and showed that the optimal index tracking portfolio has stationary tracking errors and that efficient long-short hedge strategies can be achieved with relatively few stocks and less turnover. Alexander (2001) discussed in detail all the relevant published work in finance that used different properties of cointegration for portfolio optimization or the construction of trading strategies.

Cointegration is defined largely by how it combines the two methods for asset allocation. These are *strategic asset allocation*, which selects a *target asset allocation* or *index*, and *tactical asset allocation*, which selects securities that are outside the target. Strategic asset allocation is a process of selecting the appropriate benchmark for a portfolio. For example, pension plans regularly go through such a benchmark selection in order to establish their investment policies. Also, tactical asset allocation is usually associated with active portfolio management. It may address how one should maintain the allocation of sixty percent stocks and forty percent bonds over time.

Cointegration has been shown to affect strategic asset allocation. In other words, the decision about optimal portfolio mix (or setting the appropriate benchmark) is influenced by the common stochastic trend between assets. Lucas (1997) presented a model where a portfolio manager maximized the expected utility of total earnings over a finite time period. The associated time-series model captured cointegrating relations among the included assets. He showed that cointegration affects strategic asset allocation, whereas error-correction mainly affects tactical asset allocation.

Using cointegration for trading or portfolio allocation assumes a long-term stochastic trend. In general, this contradicts the hypothesis that stock price returns follow a random walk. Lo & MacKinley (1988) tested the random walk hypothesis with weekly stock market returns by comparing variance estimators derived from data sampled at different frequencies. If the stock returns followed a random walk, then the variance should have grown with the square root of time. But the authors found that due largely to the behavior of small stocks, the random walk hypothesis was rejected. Additionally, they showed that the autocorrelations of individual securities were generally negative and that the autocorrelations of equally and positively weighted CRSP indices were positive.

Technical trading strategies that explore short-term market inefficiencies have also been widely used by hedge fund managers. Gatev et al. (2006) showed how to construct a "pairs trading" strategy whose profits typically exceeded conservative transaction-cost estimates. The authors linked the profitability of this strategy to the presence of a common factor in the returns – different from conventional risk measures.

Lo & MacKinley (1990) described a contrarian strategy that sold "winners" and bought "losers" with a positive expected return, and that despite negative autocorrelations in individual stock returns, weekly portfolio returns were strongly and positively autocorrelated. Brown & Jennings (1989) showed that technical analysis (or use of past prices to infer private information) had value in a model where prices were not fully revealing and where traders had rational conjectures about the relationship of prices to signals. Conrad & Kaul (1998) analyzed a wide range of trading strategies in use during 1926–1989. They showed that momentum and contrarian strategies were equally likely to succeed. Additionally, they found that the cross-sectional variation in mean-returns of individual securities included in the strategies was an important determinant of their profitability. Conrad & Kaul (1998) stated that cross-sectional variation may account for the profitability of momentum strategies and that it could be responsible for some of the profits from price reversals to long-horizon contrarian strategies. Cooper (1999) showed evidence of predictability by filtering lagged returns and lagged volume information to uncover weekly overreaction profits on large cap stocks.

Galenko, Popova & Popova (2011) derived new properties of cointegrated time series and constructed a trading strategy using these properties. We use their theoretical results and show a practical implementation of the strategy by using four world stock market indices. In-sample (as well as out-of-sample) results show that long-term dependencies can be exploited in a variant of "pairs trading" strategy. The results do not vanish when an out-of-sample test is performed. The problems with in-sample overfitting were well documented by Bossaerts & Hillion (1999), Pesaran & Timmermann (1995), Cooper (1999), and Conrad et al. (2003). Additionally, the "data snooping" problem and its relation to out-of-sample tests (as documented by Conrad et al. 2003, Cooper & Gulen 2006, Sullivan et al. 1999) is not a problem for our trading strategy because it is based on a theoretical relationship for cointegrated time series.

CONSTRUCTING PORTFOLIOS OF COINTEGRATED ASSETS

Galenko, Popova & Popova (2011) proved new properties for cointegrated time series and showed how to use them in order to construct a trading strategy. Their theoretical results showed that such a strategy will always have a positive expected return. To achieve that only requires a set of cointegrated time series. The reasoning behind their portfolio construction method is fairly simple. The cointegration relations between time series imply that the time series are bound together and will fairly quickly reconverge after any period of divergence. This can be formally expressed as follows.

A stochastic process Y_t is stationary if its first and second moments are time invariant: in particular, if $\mathrm{E}[Y_t] = \mu, \forall t$, and $\mathrm{E}[(Y_t - \mu)(Y_{t-h} - \mu)^T] = \Gamma_Y(\mathrm{h}) = \Gamma_Y(-\mathrm{h})^{\mathrm{T}}, \forall \mathrm{t}, \mathrm{h} = 0, 1, 2, \cdots$, where μ is a vector of finite mean terms, and $\Gamma_Y(\mathrm{h})$ is a matrix of finite covariances. Such a process is known as integrated *of order 0* and denoted by I(0). A univariate process is called integrated *of order d*, $\mathrm{I}(d)$, if in its original form it is non-stationary but becomes stationary after differencing d times. If all elements of the vector X_t, for $t = 1, 2, \cdots, N$, are $\mathrm{I}(1)$, and there exists a vector b such that $b^T X_t$ is $\mathrm{I}(0)$, then the vector process X_t is said to be cointegrated and b is called the cointegrating vector. For example, two time series X and Y are cointegrated if X, Y are I(1), and there exists a scalar b such that $Z = X - bY$ is I(0).

Assume that we have N assets (stocks, stock indexes, etc.). Denote the vector of the asset prices by $P_t = \{P_t^1, \cdots, P_t^N\}$. Write each of its elements as $P_t^i = P_0^i e^{\sum_{j=0}^{t} r_j^i}$, $i = 1, \cdots, N$, where $r = \{r_t^1, \cdots, r_t^N\}$ are the continuously compounded asset returns and $P_0^1, \cdots, P_0^N$ are the initial prices. (Without loss of generality, we can assume that $P_0^1 = \cdots = P_0^N = 1$.) Then, the log-prices can be written as $lnP_t^i = lnP_0^i + \sum_{j=0}^{t} r_j^i, i = 1, \cdots, N$.

Denote the corresponding cointegrating vector by $b = (b^1, \cdots, b^N)$. By the definition of cointegration, the resulting time series $Y_t = \sum_{i=1}^{N} b^i \, lnP_t^i$ will be stationary and integrated of order 0. Define $Z_t = Y_t - Y_{t-1} = \sum_{i=1}^{N} b^i \, r_t^i$.

Galenko, Popova & Popova (2011) proved the following result: *Consider a trading strategy where each time period, we buy* $-b^i \sum_{p=1}^{\infty} Z_{t-p}$ *value of asset* $i, i = 1, 2, \dots, N$, *and sell it in the next time period. Denote by* π_t, *the profit of the described strategy. Then* $\pi_t = -\sum_{p=1}^{\infty} Z_{t-p} \, Z_t$ *and* $[\pi_t] = \frac{VarZ_t}{2} > 0$.

With the above theoretical results in mind, we propose the following trading strategy:

- Step 1: using historical data, estimate the cointegration vector b.
- Step 2: using the estimated cointegration vector $\tilde{b}$ and historical data, construct $\tilde{Z}_t$ - realizations of the process $Z_t = \sum_{i=1}^{N} b^i \, r_t^i$.
- Step 3: compute the final sum $\sum_{p=1}^{P} \tilde{Z}_{t-p+1}$, where p is a parameter.
- Step 4: partition the assets into two sets and S (depending on values of $\tilde{b}$.).
- Step 5: buy (depending on which set the asset belongs to) the following number of shares (round down to get integer number of shares):

$$-\frac{\tilde{b}^i C sign\left(\sum_{p=1}^{P} \tilde{Z}_{t-p+1}\right)}{P_t^i \sum_{j \in L} \tilde{b}^j}, i \in L$$
$$\frac{\tilde{b}^i C sign\left(\sum_{p=1}^{P} \tilde{Z}_{t-p+1}\right)}{P_t^i \sum_{j \in S} \tilde{b}^j}, i \in S$$

- Step 6: close all the open positions the following trading day.
- Step 7: update the historical data set.

- Step 8: if it is time to re-estimate the cointegration vector (which happens every 22 trading days), go to step 1, otherwise go to step 2.

The next section describes the procedures used to test the portfolio and presents the empirical results.

EMPIRICAL RESULTS

To test the proposed portfolio construction, we used historical data for four equity indices: AEX, DAX, CAC, and FTSE. AEX, the best-known index of Euronext Amsterdam, is made up of the 25 most active securities in the Netherlands. This index provides a fair representation of the Dutch economy. DAX 30 (Deutsche Aktien Xchange 30) is a blue chip stock market index consisting of the 30 major German companies trading on the Frankfurt Stock Exchange. CAC 40, which takes its name from Paris Bourse's early automation system Cotation Assiste en Continu (Continuous Assisted Quotation), is a French stock market index representing a capitalization-weighted measure of the 40 most significant values among the 100 highest market caps on the Paris Bourse. FTSE 100 is a share index of the 100 most highly capitalized companies listed on the London Stock Exchange.

These data were used to conduct several backtests. The process of backtesting uses historical data to determine the hypothetical result of an investment strategy during the period the data cover. Such procedures are commonly used by financial professionals when a particular trading rule/portfolio is suggested for actual trading.

There are two types of backtesting: in-sample and out-of-sample. In-sample backtesting estimates the parameters of the portfolio by including historical data for time periods after the investment decision would have been made. Out-of-sample backtesting uses only the information available as of the moment of the investment decision, thus making it more realistic than in-sample backtesting.

The number of days used to estimate the parameters is called a window and is denoted by W. For example, a window size of 1000 ($W = 1000$) means that the last 1000 daily observations were used for estimation.

Realizations from 01/02/1996 to 12/28/2006 were used for AEX, DAX, CAC, and FTSE indices. Missing data due to differences in working days in different countries were filled using one of the SPLUS FinMetrics functions that interpolates a missing value using a spline. We also used the SPLUS vector autoregression estimation function to get the order of the autoregressive process and its Johansen (1995) rank test to estimate the cointegration vector. Trading for all tests started on 11/06/2001 and ended on 12/28/2006. Transaction cost per share was set to 1 cent. For out-of-sample testing, the cointegration vector b was re-estimated every 22 days. The value of the long/short positions each day was set to 10,000,000 dollars (from which the value may vary due to rounding).

In-Sample Results

To estimate the parameters for the in-sample test, we used all available historical data. Results from this test for different values of P (the lag parameter) can be found in Tables 1 and 2, where Table 1 shows the results without transaction costs and Table 2 shows the statistics after transaction costs of 1 cent per share. The long-term relationship among the four indices is estimated and described by the following cointegration vector:

$$Z = 3.69\,AEX - 4.66\,CAC + 13.57\,DAX - 21.49\,FTSE$$

The resulting process Z is $\mathrm{I}(0)$.

TABLE 1
IN-SAMPLE TEST RESULTS WITHOUT TRANSACTION COSTS

Performance Measures / Lag Parameter *P*	10	20	25	30	40
Best Day	6.09%	6.09%	6.09%	6.09%	6.09%
Worst Day	-4.64%	-4.06%	-4.06%	-4.06%	-4.06%
Percentage of Up Days	51.14%	52.50%	52.35%	53.41%	53.03%
Percentage of Down Days	48.86%	47.50%	47.65%	46.59%	46.97%
Average Daily Gain	0.57%	0.59%	0.60%	0.60%	0.59%
Standard Dev. of Positive Returns	9.97%	10.32%	10.58%	10.39%	10.20%
Average Daily Loss	-0.58%	-0.56%	-0.55%	-0.55%	-0.56%
Standard Dev. of Negative Returns	9.81%	9.40%	9.06%	9.28%	9.53%
Annual Return	2.83%	10.10%	12.90%	15.43%	11.70%
Standard Dev. of Daily Returns	13.47%	13.46%	13.45%	13.44%	13.45%
Sharpe Ratio	0.21	0.75	0.96	1.15	0.87
Sortino Ratio	0.29	1.07	1.42	1.66	1.23
Skewness	0.12	0.36	0.48	0.39	0.32
Kurtosis	5.30	5.26	5.23	5.25	5.27
Average Run Down (days)	2	2	2	2	2
Standard Dev. of Run Down (days)	1	1	1	1	1
Max Run Down (days)	8	8	9	8	8
Total Return	14.81%	52.91%	67.60%	80.82%	61.30%
Days Traded	1320	1320	1320	1320	1320

TABLE 2
IN-SAMPLE TEST RESULTS WITH TRANSACTION COSTS

Performance Measures / Lag Parameter *P*	**10**	**20**	**25**	**30**	**40**
Best Day	6.09%	6.09%	6.09%	6.09%	6.09%
Worst Day	-4.64%	-4.06%	-4.06%	-4.06%	-4.06%
Percentage of Up Days	50.91%	52.27%	52.05%	53.18%	52.88%
Percentage of Down Days	49.09%	47.73%	47.95%	46.82%	47.12%
Average Daily Gain	0.57%	0.59%	0.60%	0.60%	0.59%
Standard Dev. of Positive Returns	9.97%	10.32%	10.59%	10.39%	10.21%
Average Daily Loss	-0.58%	-0.56%	-0.55%	-0.55%	-0.56%
Standard Dev. of Negative Returns	9.81%	9.40%	9.06%	9.28%	9.53%
Annual Return	2.33%	9.61%	12.41%	14.94%	11.21%
Standard Dev. of Daily Returns	13.47%	13.46%	13.45%	13.44%	13.45%
Sharpe Ratio	0.17	0.71	0.92	1.11	0.83
Sortino Ratio	0.24	1.02	1.37	1.61	1.18
Skewness	0.12	0.36	0.48	0.39	0.32
Kurtosis	5.30	5.26	5.23	5.25	5.27
Average Run Down (days)	2	2	2	2	2
Standard Dev. of Run Down (days)	1	1	1	1	1
Max Run Down (days)	8	8	9	8	8
Total Return	12.22%	50.32%	65.00%	78.23%	58.71%
Days Traded	1320	1320	1320	1320	1320

Tables 1 and 2 report various performance measures: best and worst days, percentage of up and down days, average daily gains and losses, volatility of positive and negative returns, Sharpe and Sortino ratios, and median, skewness, and kurtosis of the daily returns, as well as the average, standard deviation, and maximum run down (run down is the number of consecutive days with negative returns).

As the value of the lag parameter changes from 10 to 40 days, the performance statistics change as well. The best results in terms of the corresponding Sharpe and Sortino ratios are for $P = 30$ days, yielding a Sharpe ratio of 1.15 and a Sortino ratio of 1.66. The annual return is 15.43%, with a volatility of 13.44%. The maximum number of days with consecutive negative returns is 8, and the average run down days is 2. The performance statistics are based on 1320 trading days. The total return for the covered period is 80.82% without transaction costs and 78.23% with transaction costs.

Table 2 shows in-sample results after transaction costs. Although the results slightly deteriorate, the pattern stays the same. Therefore, we will report results without transaction costs, as those costs vary in real life. Figure 1 shows the profit and loss (P&L) plot for the strategy, as well as the four market indices. The trading strategy performs better than a simple buy-and-hold of the individual indices.

FIGURE 1
TOTAL RETURN FOR THE IN-SAMPLE BACKTEST VS. THE FOUR INDICES

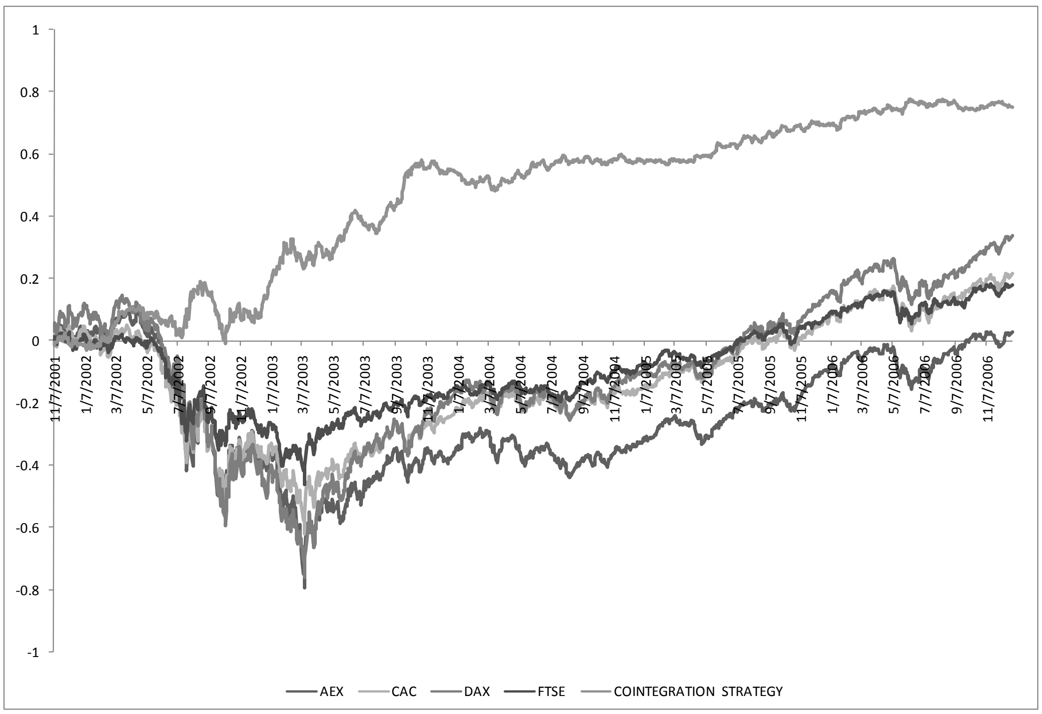


Table 3 shows the correlation between the portfolio's and each index's daily returns. The results show that the performance of the portfolio is almost neutral or slightly positively correlated with each individual index.

TABLE 3
CORRELATIONS BETWEEN THE STRATEGY IN-SAMPLE RETURNS AND THE INDIVIDUAL INDICES

Correlations	AEX	CAC	DAX	FTSE	Strategy
AEX	1.00				
CAC	0.94	1.00			
DAX	0.84	0.87	1.00		
FTSE	0.85	0.86	0.75	1.00	
Strategy	-0.06	-0.04	-0.01	-0.04	1.00

Out-of-Sample Results

An out-of-sample test is the only available tool to truly test how a portfolio would have performed if traded in the past. One of its drawbacks is that we are testing the performance of the portfolio on one sample path only. Another drawback is that we face the Uncertainty Principle of Heisenberg (see Bernstein 2007), whose practical interpretation is that one cannot measure the risk of an asset on the basis of past data alone. Entering the first trade effectively changes the market, which means that any type of backtest will not be a perfect predictor of future performance.

We designed several out-of-sample backtests by changing the values of the critical parameters. All of them start on 11/06/2001. The amount of historical data used (or the size of the window) varies from 1000 to 1500 days. For the first set of out-of-sample tests, we used a sliding window to compute the parameters. Every 22 days, the parameters of the cointegration vector were re-estimated using the last 1000, 1250, and 1500 days. A second set of out-of-sample backtests used a cumulative window, where the initial set of windows was 1000, 1250, and 1500 days. With every new trading day, the historical window used increased by one day. Tables 4, 5, and 6 show performance measures for the first set of out-of-sample tests with sliding windows 1000, 1250, and 1500, respectively.

TABLE 4
OUT-OF-SAMPLE TEST WITHOUT TRANSACTION COST,
SLIDING WINDOW SIZE, 1000 DAYS

Performance Measures / Lag Parameter *P*	**10**	**20**	**25**	**30**	**40**
Best Day	3.77%	3.77%	3.84%	3.84%	3.84%
Worst Day	-4.05%	-4.05%	-4.05%	-4.05%	-4.05%
Percentage of Up Days	50.91%	52.73%	53.26%	52.42%	50.91%
Percentage of Down Days	49.09%	47.27%	46.74%	47.58%	49.09%
Average Daily Gain	0.48%	0.50%	0.52%	0.51%	0.49%
Standard Dev. of Positive Returns	7.46%	7.59%	7.99%	7.61%	7.51%
Average Daily Loss	-0.49%	-0.47%	-0.45%	-0.46%	-0.49%
Standard Dev. of Negative Returns	7.87%	7.75%	7.23%	7.70%	7.82%
Annual Return	2.00%	9.67%	16.31%	11.62%	2.21%
Standard Dev. of Daily Returns	10.87%	10.86%	10.82%	10.85%	10.87%
Sharpe Ratio	0.18	0.89	1.51	1.07	0.20
Sortino Ratio	0.25	1.25	2.26	1.51	0.28
Skewness	-0.14	-0.12	0.20	-0.13	-0.15
Kurtosis	4.34	4.38	4.33	4.40	4.34
Average Run Down (days)	2	2	2	2	2
Standard Dev. of Run Down (days)	1	1	1	1	1
Max Run Down (days)	9	7	7	8	10
Total Return	10.50%	50.67%	85.45%	60.86%	11.57%
Days Traded	1320	1320	1320	1320	1320

TABLE 5
OUT-OF-SAMPLE TEST WITHOUT TRANSACTION COSTS,
SLIDING WINDOW SIZE 1250 DAYS

Performance Measures / Lag Parameter *P*	**10**	**20**	**25**	**30**	**40**
Best Day	3.78%	3.78%	3.86%	3.86%	3.86%
Worst Day	-3.99%	-3.99%	-3.99%	-3.99%	-3.99%
Percentage of Up Days	50.23%	52.65%	52.73%	52.65%	52.50%
Percentage of Down Days	49.77%	47.35%	47.27%	47.35%	47.50%
Average Daily Gain	0.48%	0.49%	0.50%	0.49%	0.47%

Standard Dev. of Positive Returns	7.55%	7.50%	7.88%	7.50%	7.40%
Average Daily Loss	-0.46%	-0.46%	-0.44%	-0.45%	-0.47%
Standard Dev. of Negative Returns	7.64%	7.68%	7.22%	7.69%	7.80%
Annual Return	3.20%	10.07%	14.82%	10.38%	6.43%
Standard Dev. of Daily Returns	10.66%	10.64%	10.62%	10.64%	10.65%
Sharpe Ratio	0.30	0.95	1.40	0.98	0.60
Sortino Ratio	0.42	1.31	2.05	1.35	0.83
Skewness	-0.06	-0.11	0.21	-0.11	-0.15
Kurtosis	4.74	4.79	4.73	4.80	4.77
Average Run Down (days)	2	2	2	2	2
Standard Dev. of Run Down (days)	1	1	1	1	1
Max Run Down (days)	8	7	10	7	10
Total Return	16.79%	52.76%	77.65%	54.38%	33.69%
Days Traded	1320	1320	1320	1320	1320

TABLE 6
OUT-OF-SAMPLE TEST WITHOUT TRANSACTION COST, SLIDING WINDOW SIZE 1500 DAYS

Performance Measures / Lag Parameter *P*	**10**	**20**	**25**	**30**	**40**
Best Day	3.79%	3.79%	3.92%	3.92%	3.92%
Worst Day	-3.92%	-3.92%	-3.90%	-3.90%	-3.90%
Percentage of Up Days	50.91%	54.09%	53.94%	53.56%	52.65%
Percentage of Down Days	49.09%	45.91%	46.06%	46.44%	47.35%
Average Daily Gain	0.48%	0.48%	0.50%	0.48%	0.49%
Standard Dev. of Positive Returns	7.94%	7.93%	8.21%	7.69%	8.11%
Average Daily Loss	-0.46%	-0.45%	-0.44%	-0.46%	-0.45%
Standard Dev. of Negative Returns	7.69%	7.68%	7.31%	7.97%	7.47%
Annual Return	4.99%	13.54%	16.41%	10.01%	12.23%
Standard Dev. of Daily Returns	10.81%	10.78%	10.76%	10.79%	10.79%
Sharpe Ratio	0.46	1.26	1.52	0.93	1.13
Sortino Ratio	0.65	1.76	2.24	1.26	1.64
Skewness	0.05	0.00	0.27	-0.13	0.21
Kurtosis	5.20	5.27	5.19	5.27	5.19
Average Run Down (days)	2	2	2	2	2
Standard Dev. of Run Down (days)	1	1	1	1	1
Max Run Down (days)	8	8	10	8	10
Total Return	26.14%	70.91%	85.93%	52.42%	64.04%
Days Traded	1320	1320	1320	1320	1320

Relative to the in-sample results, the strategy's performance did not change dramatically. For values of the lag parameter equal to 25 or 30 days, the Sharpe ratio varied from 0.93 to 1.51, while the Sortino ratio varied from 1.26 to 2.26. The best results for these ratios were obtained for a sliding window of 1000 days and a lag parameter of 25 days. For this case, the Sharpe ratio was 1.35 and the Sortino ratio was 1.98.

Figure 2 presents the total return of the strategy (fixed window setup) as a function of the lag P and the window size W. Figure 3 is the same for the cumulative window case. These figures indicate that the performance of the strategy is influenced more by lag parameter than by window size.

FIGURE 2
TOTAL RETURN FOR OUT-OF-SAMPLE TEST AS A FUNCTION OF THE LAG PARAMETER AND WINDOW SIZE, FIXED CASE, NO TRANSACTION COSTS

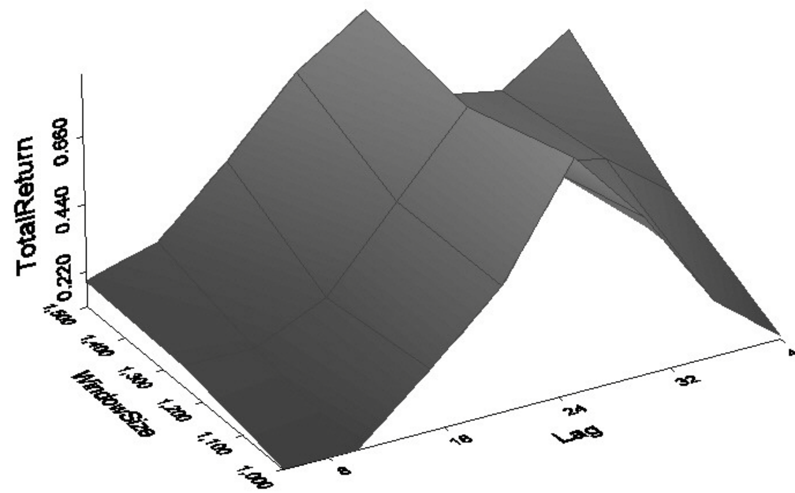


FIGURE 3
TOTAL RETURN FOR OUT-OF-SAMPLE AGGREGATE TEST AS A FUNCTION OF THE LAG PARAMETER AND WINDOW SIZE, NO TRANSACTION COSTS

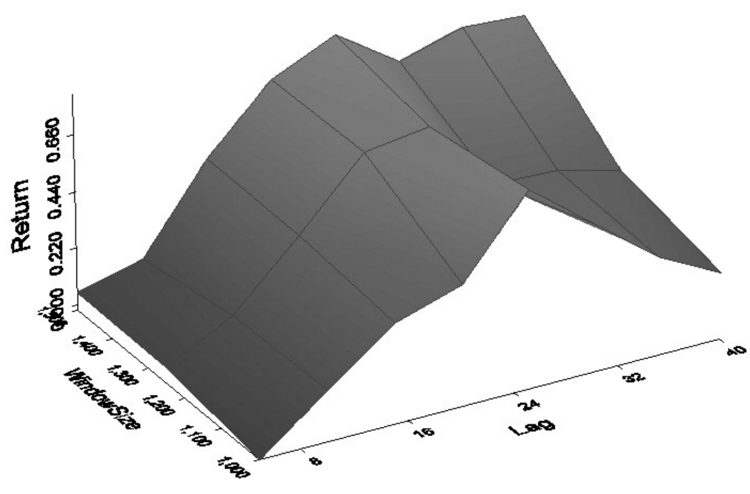


Figure 4 shows the profit and loss for the strategy when the lag parameter is 25 days and the sliding window is 1000 days. Similar to the in-sample test, the strategy performs better than buy-and-hold of the four indices.

FIGURE 4
TOTAL RETURN FOR THE OUT-OF-SAMPLE BACKTEST (LAG 25, WINDOW SIZE 1000) VS. THE FOUR INDICES

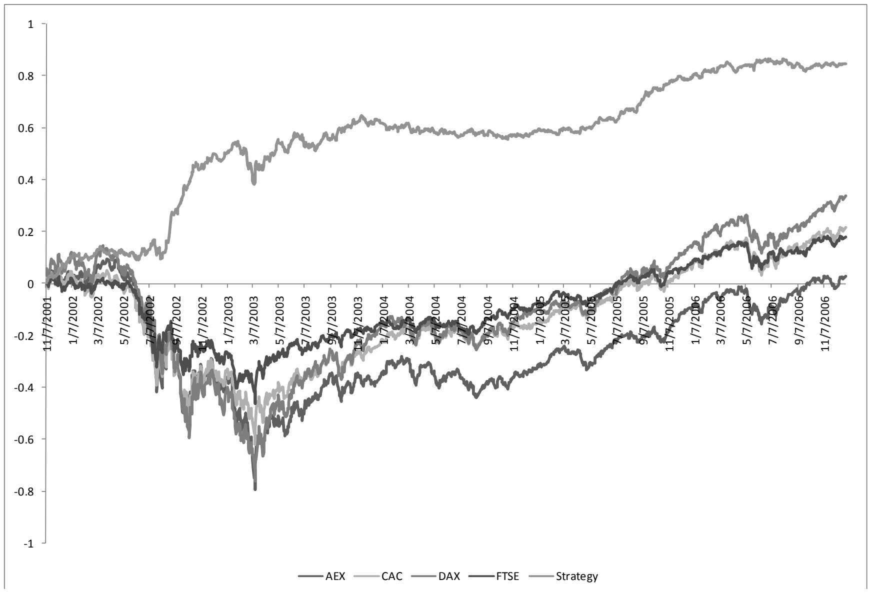


Table 7 shows the correlations of the strategy with the individual indices. Similar to the in-sample test, the strategy is almost neutral to the four market indices.

TABLE 7
CORRELATIONS BETWEEN THE STRATEGY OUT-OF-SAMPLE RETURNS AND THE INDIVIDUAL INDICES.

Correlations	AEX	CAC	DAX	FTSE	Strategy
AEX	1.00				
CAC	0.94	1.00			
DAX	0.84	0.87	1.00		
FTSE	0.85	0.86	0.75	1.00	
Strategy	-0.02	-0.03	0.01	-0.04	1.00

Figure 5 shows the time series of the cointegration vector.

Figures 4 and 5, if analyzed together, show that the values of the cointegration vector did not change substantially from 11/06/2001 to 3/25/2003. During this period, the strategy performed extremely well, gaining almost 50% while all four indices lost approximately 60% of their value. The cointegration vector changed dramatically on 3/25/2003 — a couple of days after the start of the Iraq war. The next major adjustment in the parameters of the cointegration vector occurred on 8/16/2005, almost one month after the terrorist attack in London.

Tables 8, 9, and 10 show performance measures for the second set of out-of-sample tests. These tables depict cumulative windows starting with 1000, 1250, and 1500 days, respectively. The best results in terms of Sharpe and Sortino ratios were obtained for lag parameter of 25 days and cumulative window starting at 1500 days, yielding Sharpe ratio of 1.39 and Sortino ratio of 2.02.

FIGURE 5
TIME PLOT OF THE ESTIMATED COINTEGRATION VECTOR

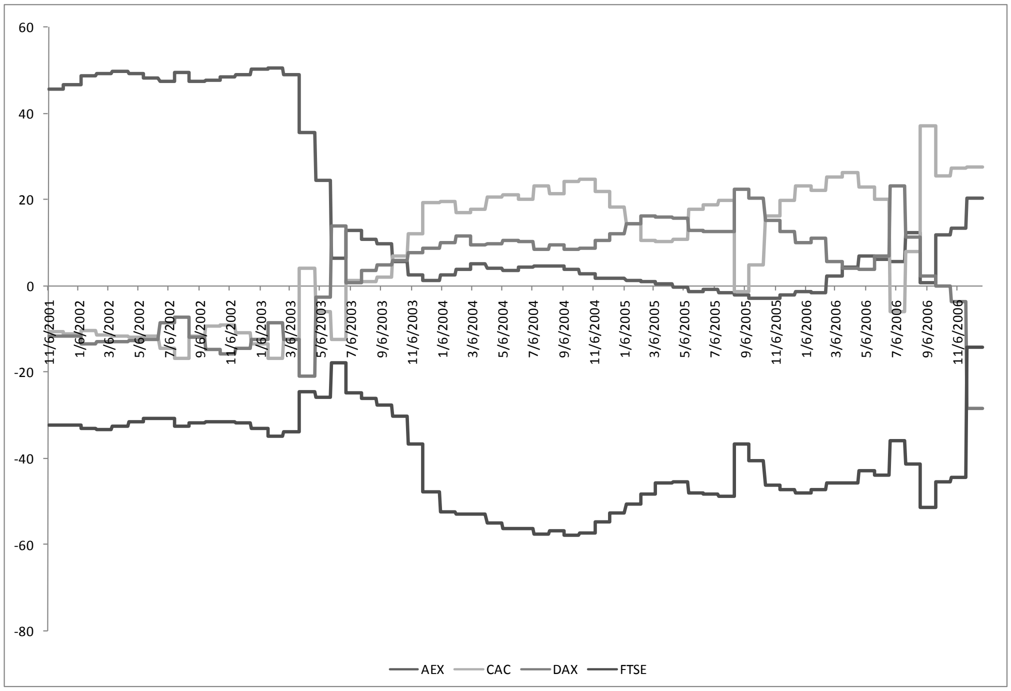


TABLE 8
OUT-OF-SAMPLE TEST WITHOUT TRANSACTION COST, CUMULATIVE WINDOW, STARTING WITH WINDOW SIZE 1000 DAYS

Performance Measures / Lag Parameter *P*	**10**	**20**	**25**	**30**	**40**
Best Day	3.80%	3.80%	3.88%	3.88%	3.88%
Worst Day	-4.03%	-4.03%	-4.03%	-4.03%	-4.03%
Percentage of Up Days	50.30%	52.12%	52.65%	53.11%	51.52%
Percentage of Down Days	49.70%	47.88%	47.35%	46.89%	48.48%
Average Daily Gain	0.47%	0.47%	0.49%	0.47%	0.46%
Standard Dev. of Positive Returns	7.41%	7.44%	7.89%	7.42%	7.46%
Average Daily Loss	-0.45%	-0.45%	-0.43%	-0.45%	-0.46%
Standard Dev. of Negative Returns	7.71%	7.69%	7.15%	7.72%	7.68%
Annual Return	3.36%	8.51%	14.13%	10.49%	4.56%
Standard Dev. of Daily Returns	10.52%	10.51%	10.49%	10.50%	10.52%
Sharpe Ratio	0.32	0.81	1.35	1.00	0.43
Sortino Ratio	0.44	1.11	1.98	1.36	0.59
Skewness	-0.14	-0.15	0.22	-0.17	-0.10
Kurtosis	5.13	5.17	5.11	5.20	5.13
Average Run Down (days)	2	2	2	2	2
Standard Dev. of Run Down (days)	1	1	1	1	1
Max Run Down (days)	8	9	8	8	8
Total Return	17.59%	44.57%	74.01%	54.95%	23.89%
Days Traded	1320	1320	1320	1320	1320

TABLE 9
OUT-OF-SAMPLE TEST WITHOUT TRANSACTION COST, CUMULATIVE WINDOW, STARTING WITH WINDOW SIZE 1250 DAYS

Performance Measures / Lag Parameter *P*	**10**	**20**	**25**	**30**	**40**
Best Day	3.76%	3.76%	3.88%	3.88%	3.88%
Worst Day	-3.93%	-3.93%	-3.93%	-3.93%	-3.93%
Percentage of Up Days	50.23%	53.64%	53.26%	52.88%	52.27%
Percentage of Down Days	49.77%	46.36%	46.74%	47.12%	47.73%
Average Daily Gain	0.49%	0.49%	0.50%	0.48%	0.48%
Standard Dev. of Positive Returns	7.74%	7.67%	7.96%	7.46%	7.61%
Average Daily Loss	-0.46%	-0.46%	-0.45%	-0.47%	-0.47%
Standard Dev. of Negative Returns	7.55%	7.61%	7.25%	7.85%	7.68%
Annual Return	3.29%	12.97%	13.65%	8.33%	7.05%
Standard Dev. of Daily Returns	10.74%	10.71%	10.70%	10.73%	10.73%
Sharpe Ratio	0.31	1.21	1.28	0.78	0.66
Sortino Ratio	0.44	1.70	1.88	1.06	0.92
Skewness	0.05	-0.07	0.22	-0.17	-0.06
Kurtosis	4.91	4.99	4.90	4.96	4.94
Average Run Down (days)	2	2	2	2	2
Standard Dev. of Run Down (days)	1	1	1	1	1
Max Run Down (days)	8	7	7	7	7
Total Return	17.22%	67.92%	71.51%	43.65%	36.94%
Days Traded	1320	1320	1320	1320	1320

TABLE 10
OUT-OF-SAMPLE TEST WITHOUT TRANSACTION COST, CUMULATIVE WINDOW, STARTING WITH WINDOW SIZE 1500 DAYS

Performance Measures / Lag Parameter *P*	**10**	**20**	**25**	**30**	**40**
Best Day	3.91%	4.07%	4.07%	4.07%	4.07%
Worst Day	-4.37%	-4.37%	-4.37%	-4.37%	-4.37%
Percentage of Up Days	49.92%	52.80%	52.88%	53.18%	51.97%
Percentage of Down Days	50.08%	47.20%	47.12%	46.82%	48.03%
Average Daily Gain	0.50%	0.51%	0.52%	0.50%	0.52%
Standard Dev. of Positive Returns	7.92%	8.32%	8.42%	8.19%	8.68%
Average Daily Loss	-0.49%	-0.47%	-0.46%	-0.47%	-0.46%
Standard Dev. of Negative Returns	8.30%	7.85%	7.71%	8.01%	7.41%
Annual Return	1.09%	12.23%	14.43%	11.42%	12.71%
Standard Dev. of Daily Returns	11.24%	11.21%	11.20%	11.21%	11.21%
Sharpe Ratio	0.10	1.09	1.29	1.02	1.13
Sortino Ratio	0.13	1.56	1.87	1.43	1.72
Skewness	-0.17	0.13	0.17	0.03	0.39
Kurtosis	5.20	5.21	5.21	5.23	5.13
Average Run Down (days)	2	2	2	2	2
Standard Dev. of Run Down (days)	1	1	1	1	1
Max Run Down (days)	8	8	8	8	8
Total Return	5.70%	64.05%	75.61%	59.82%	66.57%
Days Traded	1320	1320	1320	1320	1320

Figure 6 shows the profit and loss for the strategy where the lag parameter is 25 days and the cumulative window starts at 1500 days. Similar to the in-sample test, the strategy performs better than buy-and-hold of the four indices.

FIGURE 6
TOTAL RETURN FOR THE OUT-OF-SAMPLE BACKTEST (LAG 25, CUMULATIVE WINDOW STARTING AT 1500) VS. THE FOUR INDICES

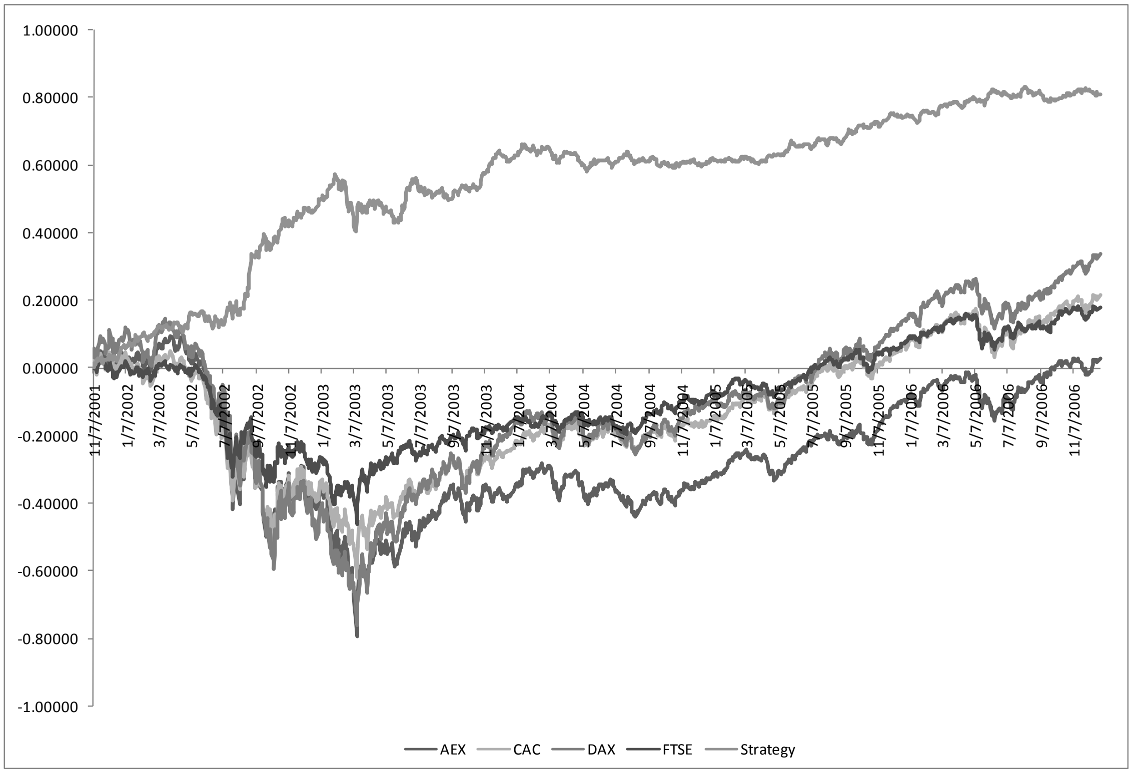


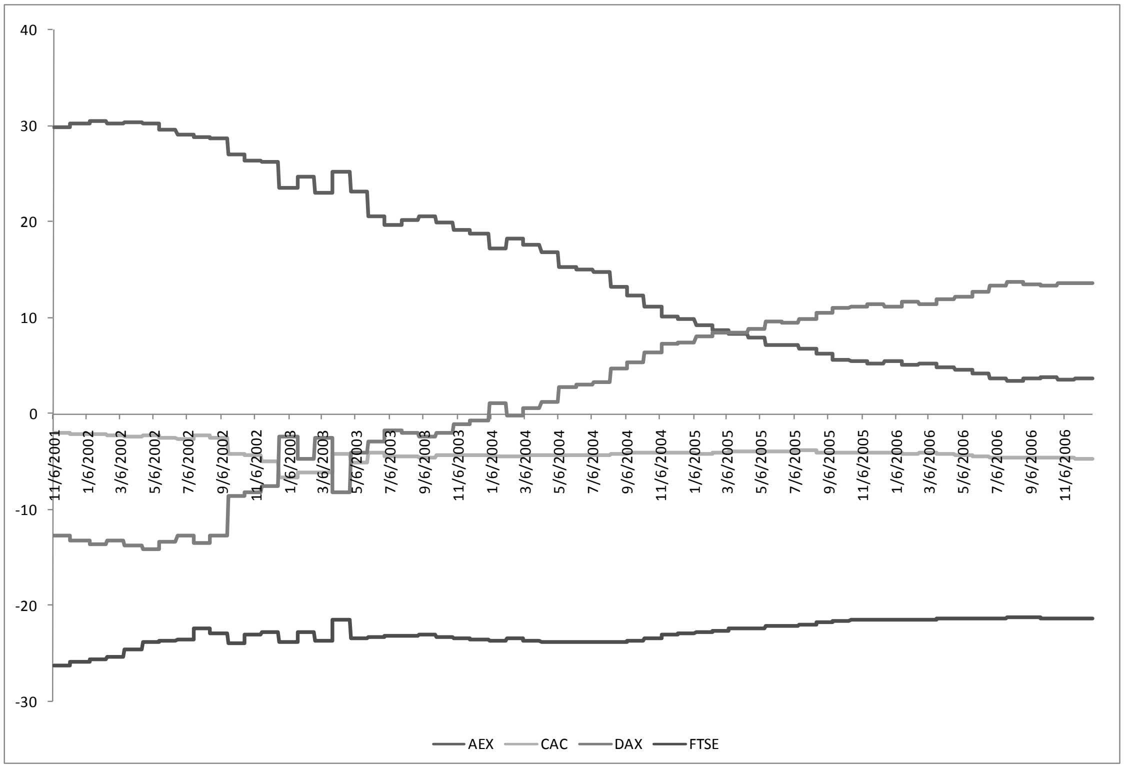
Table 11 shows the correlations of the strategy with the individual indices. Similar to the in-sample test, the strategy is almost neutral to the four market indices.

TABLE 11
CORRELATIONS OF THE DAILY RETURNS OF THE STRATEGY WITH THE DAILY RETURNS OF THE FOUR INDICES WHEN THE LAG PARAMETER IS 25 DAYS AND THE CUMULATIVE WINDOW STARTS AT 1500 DAYS

Correlations	AEX	CAC	DAX	FTSE	Strategy
AEX	1.00				
CAC	0.94	1.00			
DAX	0.84	0.87	1.00		
FTSE	0.85	0.86	0.75	1.00	
Strategy	-0.04	-0.04	0.02	-0.03	1.00

Figure 7 shows the time series of the cointegration vector.

FIGURE 7
TIME PLOT OF THE ESTIMATED COINTEGRATION VECTOR WHEN THE LAG PARAMETER IS 25 DAYS, AND THE CUMULATIVE WINDOW STARTS AT 1500 DAYS



With a cumulative window, the values of the cointegration vector are not highly variable. The only period with visible changes is at the start of the Iraq war. The results of the two designs of out-of-sample tests implied that the Iraq war profoundly affected the long-term relationship among financial assets.

Performance statistics from Tables 4 and 10 indicate that sliding window design is better than cumulative window design. This confirms Proposition 1 from Galenko, Popova & Popova (2011), which assumed that there will be a value of the lag parameter *P* for which the long-term relationship will be almost nonexistent.

CONCLUSION

We showed how to construct a daily trading strategy that explores new properties of cointegrated time series. We showed its practical implementation by using the daily closing prices of four stock market indices.

In-sample and out-of-sample tests showed that the designed portfolio significantly outperformed simple buy-and-hold of each individual index. Additionally, the time series of the cointegration vector exhibited how the portfolio adapts to events that highly stress the financial markets (like the start of the Iraq war and the terrorist attack in London).

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