Mathematical Analysis of Unemployment Benefits

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During the Great Recession, expenditures on unemployment insurance (UI) benefits increased, and the benefits were extended. This research deals with development of a mathematical model to calculate unemployment benefits. At a conceptual level, unemployment benefits can be considered as directly proportional to salary and the employment period of the worker prior to being laid off. It is also inversely proportional to factors such as other governmental benefits received in that period of unemployment. The approach in this paper presents the need to calculate the unemployment benefits to keep up with the capricious wages and changing regulations. This analysis will include state and federal government benefits. Each state should be able to adapt the new formula, so it can assess the proper baseline for calculating the unemployment benefits needed for their specific cost of living requirements.

INTRODUCTION

During the Great Recession, expenditures on unemployment insurance (UI) benefits increased substantially, from $33 billion in 2007 to $94 billion in 2012.[1] To deal with this unprecedented economic challenge from the Great Recession, the policy response also initiated the extension of unemployment benefits with benefit duration rising from the usual 26 weeks to as long as 99 weeks. The motivation was

[1] See https://www.cbo.gov/publication/43734

to provide "income support for a vulnerable group after they have lost their jobs through no fault of their own" as well as "needed support for the fragile economy after the Great Recession ended. To understand the problem in the determination of the assessing the correct unemployment benefits, this paper investigates the formulation of these benefits using mathematical and statistical concepts. In any physical problem, a mathematical model is formulated after establishing certain assumptions. However, when developing these assumptions, we need to be careful in the establishing too many assumptions because if one makes too many assumptions, the corresponding mathematical model will have little policy relevance. On the other hand, if there are too few assumptions, the mathematical model may be too complex to solve. The balance of this paper is as follows. Section 2 provides a brief overview of the related literature while section 3 provides the methodology undertaken in this paper and the empirical results. Finally, section 4 discusses the results of the analysis and concludes the paper.

BRIEF OVERVIEW OF THE RELATED LITERATURE

The seminal contributions by Moffitt (1985), Katz and Meyer (1990), Meyer (1990), and Card and Levine (2000) explored the concept of estimation of the extension of the unemployment benefits. In their analyses, they used administrative data on unemployment benefit recipients and exploited the cross-state variation in unemployment benefit extensions to measure the effect of the extensions on the hazard rate of leaving compensated unemployment. These estimates were interpreted using a partial equilibrium search model as measuring how individual search efforts respond to changes in benefits holding labor market conditions constant. Because these studies use a relatively small sample of unemployed workers who collect benefits, and the authors could not measure the impact of benefit extensions on the search effort of those who do not receive benefits, these analyses could not assess the impact of benefit extensions on overall unemployment in the United States.

Schmieder, Von Wachter, and Bender (2012) estimate the disincentive effect of unemployment benefits over the business cycle in Germany via administrative data. More specifically, they exploit a policy discontinuity based on the age of workers on the day they become unemployed. The months of unemployment benefits a worker is eligible for changes discontinuously at two age cutoffs. Using a regression discontinuity design, they are able to estimate the change in the behavioral response due to increased benefit eligibility, and how this response varies with the conditions of the business cycle. Their results reveal that a small disincentive affects overall that does not vary much with the conditions of the business cycle.

Rothstein (2011), who presents an analysis of the effects of the recent UI extensions, reports small effects of the recent UI extensions on unemployment exits, duration, and the overall unemployment rate. Rothstein (2011) estimates the partial equilibrium effects of the unemployment benefit extensions on labor market outcomes during the Great Recession via the Current Population Survey (CPS) on duration of unemployment by individuals. There has been research that delved into the persistence of high levels of unemployment beyond 2009, which was after the Great Recession. Hagedorn, Karahan, Manovskii, and Mitman (2013) argue that unemployment benefit extensions explain a large part of the persistently high level of unemployment after 2009. They examined whether more generous UI benefits result in higher wages and higher unemployment by raising the flow value of unemployment and shrinking the gap between productivity and that flow value. They compare labor markets with arguably similar conditions apart from the UI benefits regime. In their work, the markets are defined as counties and the similarity arises because they focus on pairs of adjacent counties. The difference in the UI regimes arises because the two counties are in different states and UI benefits are set at the state level and often differ across state boundaries. The authors conclude that, absent the increase in UI benefits, unemployment in 2010 would have been about 3 percent lower.

Subsequent research, e.g., Hall (2015) did not agree with the conclusions by Hagedorn, Karahan, Manovskii, and Mitman (2013) because their conclusions implies that implies that unemployment rate would have hardly risen at all absent the of financial crisis and resulting collapse of product demand.

Their work revealed that the enhancements of UI benefits were the result of the difficulties stemming from the Great Recession.

Other results from the literature implies that more generous UI benefits increase labor market tightness and the job finding rate per unit of search effort. As a consequence, the optimal level of UI will be larger than suggested by the partial equilibrium Baily Chetty formula (Chetty, 2006), as explained in Landais, Michaillat and Saez (2010). This means that temporary extensions enacted in reaction to business cycles downturns are 3 less socially costly than what a partial equilibrium representation would suggest.

Kiley (2003), Sanchez (2008), and Andersen and Svarer (2011) theoretically explore optimal UI over the business cycle in partial equilibrium job search models without savings.

METHODOLOGY AND EMPIRICAL RESULTS

The unemployment benefits really depend on two basic parameters. One is the salary that a person was making while working and the other is the period that the person worked before being laid off. While every state has a standard upper limit on the unemployment benefits, there is also an implicit minimum on the unemployment benefits. The study is done in two phases.

Phase I

In this phase, seven families were randomly chosen for this study. Tables 1-3 show the period worked in weeks, salary/week, benefits/week and the duration of the unemployment benefits that the person received. Figures 1-3 show the variation of unemployment benefits vs. salary/week for three consecutive years from 2012-2014 for the seven randomly chosen families. The intent is to find a functional relation between the parameters of unemployment benefits and the salary, unemployment benefits and duration of benefits and period of work and duration of unemployment benefits for the three consecutive years 2012-2014. Each of the function is then checked for statistical adequacy through two standard parameters. One of them is the correlation coefficient (r) and the other is the standard error of estimate ($s_{y/x}$). For an acceptable fit, $r \geq 0.8$, and $s_{y/x} < s_y$.

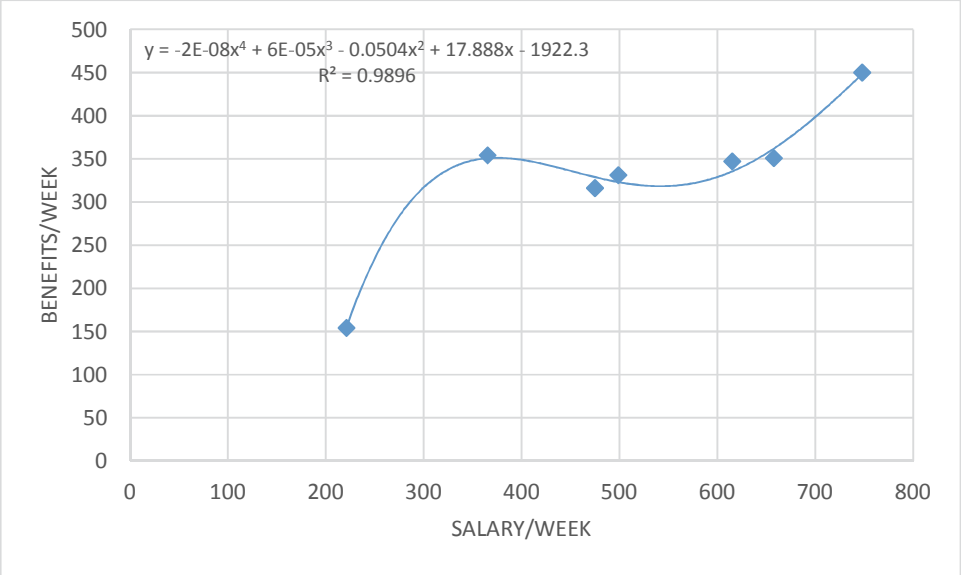
Discussion and Analysis of results

The function relations obtained are as follows for the year 2012:

TABLE 1
PERIOD WORKED IN WEEKS, SALARY WEEK, BENEFITS WEEK, AND DURATION OF UNEMPLOYMENT BENEFITS RECEIVED PER PERSON, 2012

Person	Period worked (weeks)	Salary/Week	Benefits/Week	Duration (Total Maximum) (weeks)
1	52	475	316	26
2	52	365.38	354	26
3	52	221.15	154	26
4	52	499.04	331	26
5	52	748.08	450	26
6	52	657.69	351	26
7	52	615.38	347	26

FIGURE 2
UNEMPLOYMENT BENEFITS VERSUS SALARY, 2012



The equation for the unemployment benefits versus salary is

$$y = -2E\text{-}08x^4 + 6E\text{-}05x^3 - 0.0504x^2 + 17.888x - 1922.3 \quad (1)$$

$r^2 = 0.9896$.

The adequacy check will check the correlation coefficient and apply the Chi Square test to check for normality. The adequacy check is

i. $r = 0.9947$ (> 0.8) Satisfactory

The next part of the adequacy check is the use of the Chi Square (χ^2) test to determine normality, and table 4 summarizes this test.

TABLE 2
RESULTS FROM THE CHI SQUARE TEST (χ^2)

	xj		Frequency	Relative Frequency ~f(x)	φ((xj-μ)/σ)			ej	bj	(bj-ej)²/ej
154		228	1	0.2	0.016234		0.108549	0.759845	1	0.07590273
228		302	0	0	0.108549		0.370717	2.595016	0	2.59501621
302		376	5	1	0.370717		0.717141	5.019984	5	7.9553E-05
376		450	1	0.2	0.717141		0.930389	6.51272	1	4.66626615
									Sum	7.337265

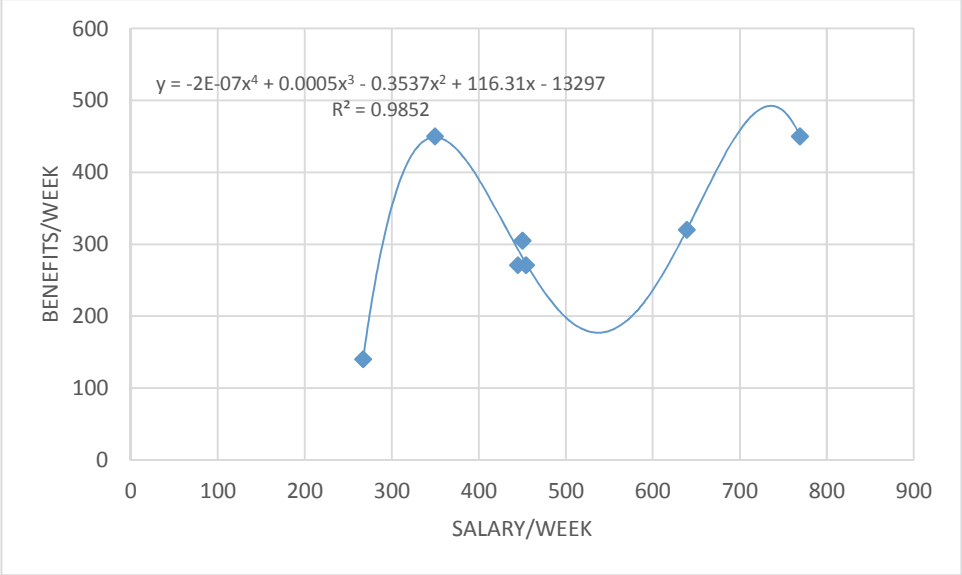
The chi-square test statistic is 7.34 with 3 df, which is less than the critical value of 9.48, and so we can conclude that there is a good fit.

TABLE 3
PERIOD WORKED IN WEEKS, SALARY WEEK, BENEFITS WEEK, AND DURATION OF UNEMPLOYMENT BENEFITS RECEIVED PER PERSON, 2013

Person	Period worked (weeks)	Salary/Week	Benefits/Week	Duration (Total Maximum) (weeks)
1	52	454.42	271	26
2	52	450.38	305	26
3	52	266.92	140	26
4	52	444.9	271	26
5	52	769.23	450	26
6	52	639	320	26
7	52	349.61	450	26

As for the unemployment benefits versus salary, it is shown in figure 2.

FIGURE 2
UNEMPLOYMENT BENEFITS VERSUS SALARY, 2013



The equation for the unemployment benefits versus salary is

$$y = -2E\text{-}07x^4 + 0.0005x^3 - 0.3537x^2 + 116.31x - 13297 \quad (2)$$

$r^2 = 0.9852$

The adequacy check will check the correlation coefficient and apply the Chi Square (χ^2) test to check for normality. The results of the adequacy check is

i. $r = 0.992572$ (> 0.8) Satisfactory

The next part of the adequacy check is the use of the Chi Square (χ^2) test to determine normality, and table 4 summarizes this test.

TABLE 4
RESULTS FROM THE CHI SQUARE TEST (χ^2)

	xj		Frequency	Relative Frequency ~f(x)	φ((xj-μ)/σ)			ej	bj	$(bj-ej)^2/ej$
140	-	217.5	1	0.2	0.040965	-	0.165909	1.161363	1	0.02242028
217.5	-	295	2	0.4	0.165909	-	0.420223	2.941563	2	0.30138434
295	-	372.5	2	0.4	0.420223	-	0.714919	5.004435	2	1.80372618
372.5	-	450	2	0.4	0.714919	-	0.909381	6.365664	2	2.99403484
									Sum	5.121566

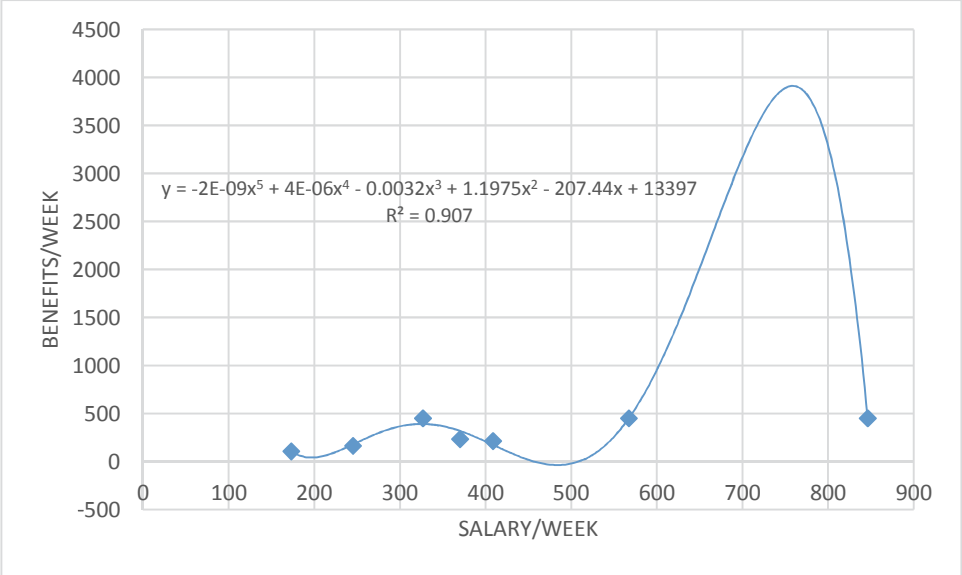
The chi-square test statistic is 5.121 with 3 df, which is less than the critical value of 9.48, and so we can conclude that there is a good fit.

TABLE 5
PERIOD WORKED IN WEEKS, SALARY WEEK, BENEFITS WEEK, AND DURATION OF UNEMPLOYMENT BENEFITS RECEIVED PER PERSON, 2014

Person	Period Worked (weeks)	Salary/Week	Benefits/Week	Duration (Total Maximum) (weeks)
1	52	370.19	231	13
2	40	245.19	163.63	13
3	52	173.07	105.88	13
4	52	408.65	211.75	13
5	52	846.15	450	19
6	52	567.31	450	14
7	40	326.92	450	17

As for the unemployment benefits versus salary, it is shown in figure 3.

FIGURE 3
UNEMPLOYMENT BENEFITS VERSUS SALARY, 2014



The estimated equation for the unemployment benefits versus salary is given as

$$y = -2E\text{-}09x^5 + 4E\text{-}06x^4 - 0.0032x^3 + 1.1975x^2 - 207.44x + 13397 \quad (3)$$

$$r^2 = 0.907$$

The results of the adequacy check are as follows:

i. r = 0.952365 (> 0.8) Satisfactory

The next part of the adequacy check is the use of the Chi Square (χ^2) test to determine normality, and table 5 summarizes this test.

TABLE 5
RESULTS FROM THE CHI SQUARE TEST (χ^2)

	xj		Frequency	Relative Frequency ~f(x)	φ((xj-μ)/σ)			ej	bj	(bj-ej)²/ej
105.88	-	191.91	2	0.4	0.087968	-	0.230728	1.615094	2	0.09172982
191.91	-	277.94	2	0.4	0.230728	-	0.452428	3.166993	2	0.43002069
277.94	-	363.97	0	0	0.452428	-	0.690543	4.833804	0	4.83380405
363.97	-	450	3	0.6	0.690543	-	0.867428	6.071993	0	6.07199285
									Sum	11.42755

The chi-square test statistic is 11.43 with 3 df, which is greater than the critical value of 9.48, and so we can conclude that there is not a good fit.

PHASE 2

In this phase, five states (Oregon, Missouri, Florida, Arizona and Ohio) were randomly picked, and the information on average weekly benefits for each of the 12 months is collected and is shown in Tables 6-10. Figures 4-6 show the histogram of these average/weekly benefits. A visual inspection shows a normal distribution, which can be verified via the chi-square test. Table 9 shows statistical analysis of the unemployment benefits data.

TABLE 6
AVERAGE WEEKLY BENEFITS FOR 12 MONTHS FOR OREGON
(Source: United States Department of Labor, Employment and Training Administration, 2015)

Week	Average Weekly Benefits
1/31/2011	$284.72
2/28/2011	$280.66
3/31/2011	$281.39
4/30/2011	$285.37
5/31/2011	$285.47
6/30/2011	$286.72
7/31/2011	$288.81
8/31/2011	$290.06
9/30/2011	$296.44
10/31/2011	$299.66
11/30/2011	$293.58
12/31/2011	$288.74
1/31/2012	$289.18
2/29/2012	$286.42
3/31/2012	$290.69
4/30/2012	$294.37
5/31/2012	$301.58
6/30/2012	$303.27
7/31/2012	$306.27
8/31/2012	$307.61
9/30/2012	$315.52
10/31/2012	$320.06
11/30/2012	$314.31
12/31/2012	$311.44
1/31/2013	$307.95
2/28/2013	$304.95
3/31/2013	$307.88
4/30/2013	$310.04
5/31/2013	$315.38
6/30/2013	$317.01
7/31/2013	$317.22
8/31/2013	$318.17
9/30/2013	$328.84
10/31/2013	$330.04
11/30/2013	$322.68
12/31/2013	$319.08
1/31/2014	$312.54

FIGURE 4
THE RELATIVE FREQUENCY FOR OREGON

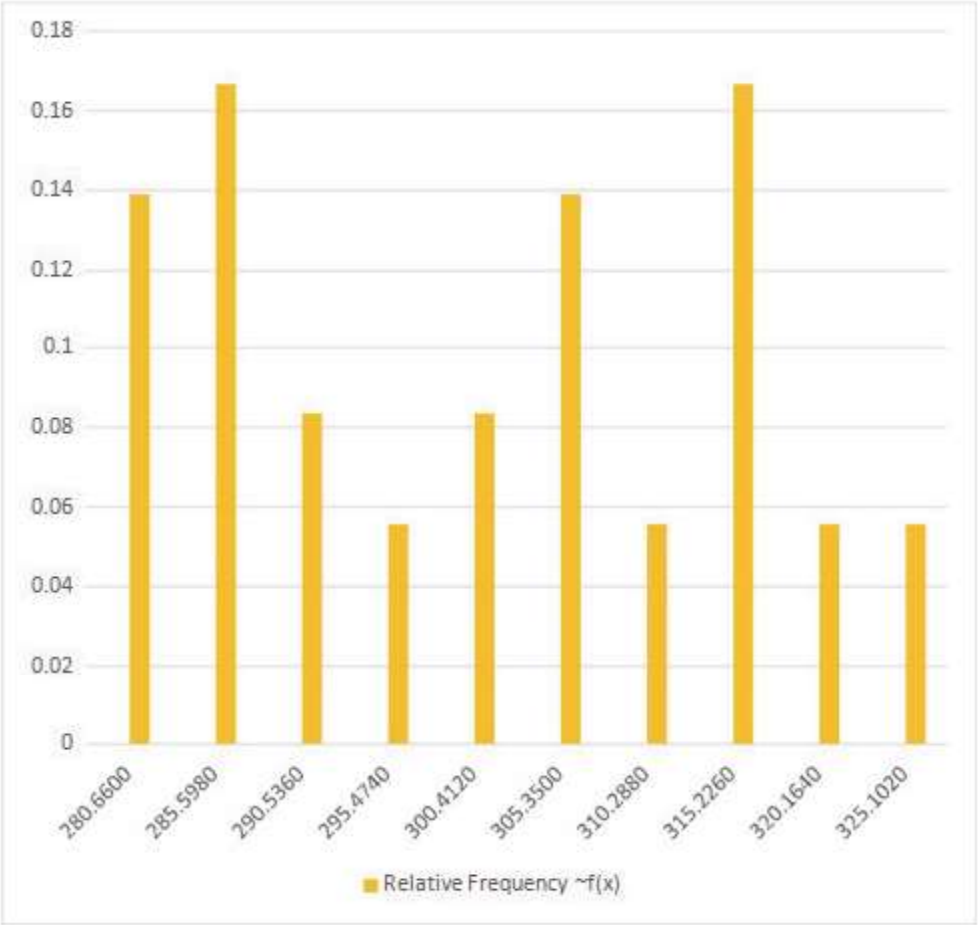


TABLE 7
AVERAGE WEEKLY BENEFITS FOR 12 MONTHS FOR MISSOURI
(Source: United States Department of Labor, Employment and Training Administration, 2015)

Week	Average Weekly Benefits	Week	Average Weekly Benefits
1/31/2011	$241.04	**10/31/2012**	$243.27
2/28/2011	$242.92	**11/30/2012**	$241.54
3/31/2011	$240.92	**12/31/2012**	$244.30
4/30/2011	$241.10	**1/31/2013**	$245.07
5/31/2011	$240.33	**2/28/2013**	$243.79
6/30/2011	$233.31	**3/31/2013**	$245.56
7/31/2011	$228.91	**4/30/2013**	$244.87
8/31/2011	$230.12	**5/31/2013**	$241.44
9/30/2011	$237.78	**6/30/2013**	$233.23
10/31/2011	$236.97	**7/31/2013**	$231.02
11/30/2011	$237.40	**8/31/2013**	$233.99
12/31/2011	$239.08	**9/30/2013**	$242.48
1/31/2012	$237.50	**10/31/2013**	$244.40
2/29/2012	$240.15	**11/30/2013**	$244.72
3/31/2012	$240.43	**12/31/2013**	$245.91
4/30/2012	$240.86	**1/31/2014**	$244.84
5/31/2012	$241.80		
6/30/2012	$234.05		
7/31/2012	$231.83		
8/31/2012	$235.63		
9/30/2012	$244.49		

FIGURE 5
THE RELATIVE FREQUENCY FOR MISSOURI

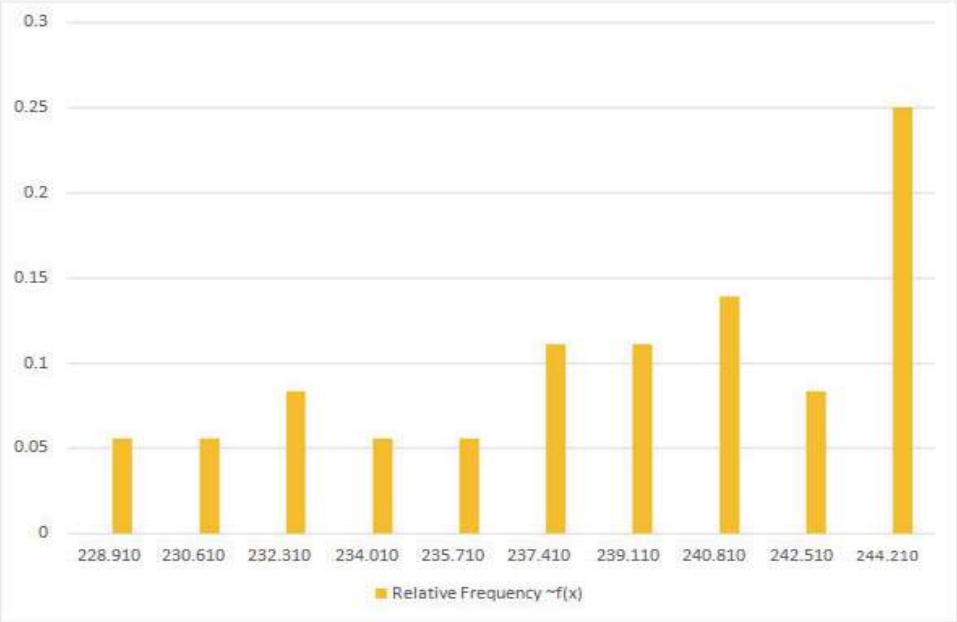


TABLE 8
AVERAGE WEEKLY BENEFITS FOR 12 MONTHS FOR FLORIDA
(Source: United States Department of Labor, Employment and Training Administration, 2015)

Week	Average Weekly Benefits	Week	Average Weekly Benefits
W1/31/2011	$229.75	**9/30/2012**	$230.63
2/28/2011	$229.74	**10/31/2012**	$230.62
3/31/2011	$230.60	**11/30/2012**	$231.28
4/30/2011	$231.61	**12/31/2012**	$232.54
5/31/2011	$232.03	**1/31/2013**	$232.53
6/30/2011	$230.42	**2/28/2013**	$233.04
7/31/2011	$228.73	**3/31/2013**	$233.62
8/31/2011	$230.26	**4/30/2013**	$234.33
9/30/2011	$230.85	**5/31/2013**	$234.75
10/31/2011	$231.34	**6/30/2013**	$233.16
11/30/2011	$231.61	**7/31/2013**	$231.13
12/31/2011	$231.80	**8/31/2013**	$230.79
1/31/2012	$231.83	**9/30/2013**	$231.75
2/29/2012	$231.76	**10/31/2013**	$236.23
3/31/2012	$232.99	**11/30/2013**	$236.73
4/30/2012	$233.80	**12/31/2013**	$237.41
5/31/2012	$233.36		
6/30/2012	$231.65		
7/31/2012	$229.49		
8/31/2012	$229.36		

FIGURE 6
THE RELATIVE FREQUENCY FOR FLORIDA

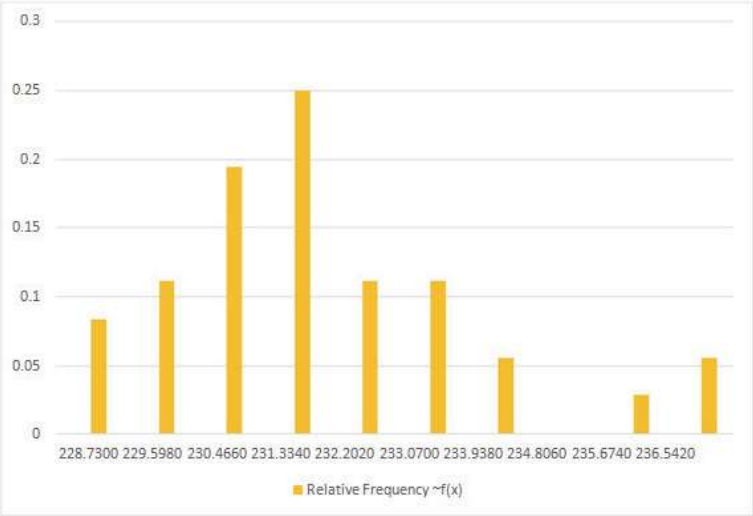


TABLE 9
AVERAGE WEEKLY BENEFITS FOR 12 MONTHS FOR ARIZONA
(Source: United States Department of Labor, Employment and Training Administration, 2015)

Week	Average Weekly Benefits	Week	Average Weekly Benefits
1/31/2011	$212.78	**9/30/2012**	$211.62
2/28/2011	$212.81	**10/31/2012**	$214.81
3/31/2011	$212.63	**11/30/2012**	$218.30
4/30/2011	$211.66	**12/31/2012**	$220.53
5/31/2011	$210.52	**1/31/2013**	$221.65
6/30/2011	$209.65	**2/28/2013**	$221.49
7/31/2011	$208.00	**3/31/2013**	$220.88
8/31/2011	$207.45	**4/30/2013**	$220.20
9/30/2011	$209.17	**5/31/2013**	$219.39
10/31/2011	$209.74	**6/30/2013**	$219.72
11/30/2011	$211.42	**7/31/2013**	$218.59
12/31/2011	$212.68	**8/31/2013**	$218.44
1/31/2012	$212.50	**9/30/2013**	$220.25
2/29/2012	$212.71	**10/31/2013**	$221.09
3/31/2012	$212.85	**11/30/2013**	$222.47
4/30/2012	$212.32	**12/31/2013**	$223.96
5/31/2012	$210.93	**1/31/2014**	$224.18
6/30/2012	$210.07		
7/31/2012	$208.19		
8/31/2012	$208.90		

FIGURE 7
THE RELATIVE FREQUENCY FOR ARIZONA

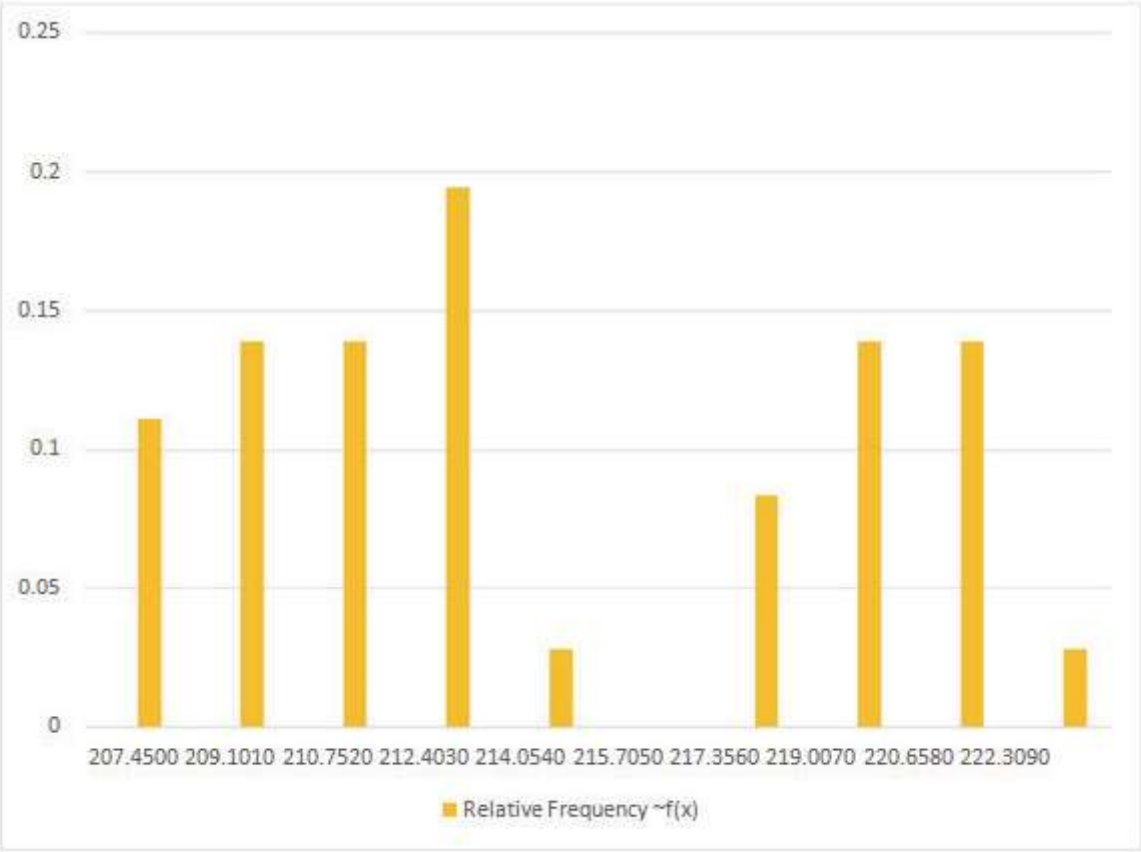


TABLE 10
AVERAGE WEEKLY BENEFITS FOR 12 MONTHS FOR CALIFORNIA
(Source: United States Department of Labor, Employment and Training Administration, 2015)

Week	Average Weekly Benefits
1/31/2011	$292.64
2/28/2011	$289.76
3/31/2011	$290.78
4/30/2011	$290.92
5/31/2011	$293.31
6/30/2011	$296.54
7/31/2011	$294.69
8/31/2011	$293.55
9/30/2011	$294.86
10/31/2011	$294.76
11/30/2011	$292.98
12/31/2011	$291.88
1/31/2012	$293.30
2/29/2012	$292.21
3/31/2012	$291.54
4/30/2012	$291.98
5/31/2012	$295.25
6/30/2012	$296.93
7/31/2012	$296.80
8/31/2012	$296.82
9/30/2012	$297.56
10/31/2012	$296.82
11/30/2012	$296.32
12/31/2012	$296.14
1/31/2013	$298.87
2/28/2013	$296.19
3/31/2013	$295.42
4/30/2013	$296.88
5/31/2013	$300.95
6/30/2013	$303.59
7/31/2013	$303.13
8/31/2013	$302.67
9/30/2013	$304.50
10/31/2013	$304.65
11/30/2013	$303.16
12/31/2013	$301.93

FIGURE 8
THE RELATIVE FREQUENCY FOR CALIFORNIA

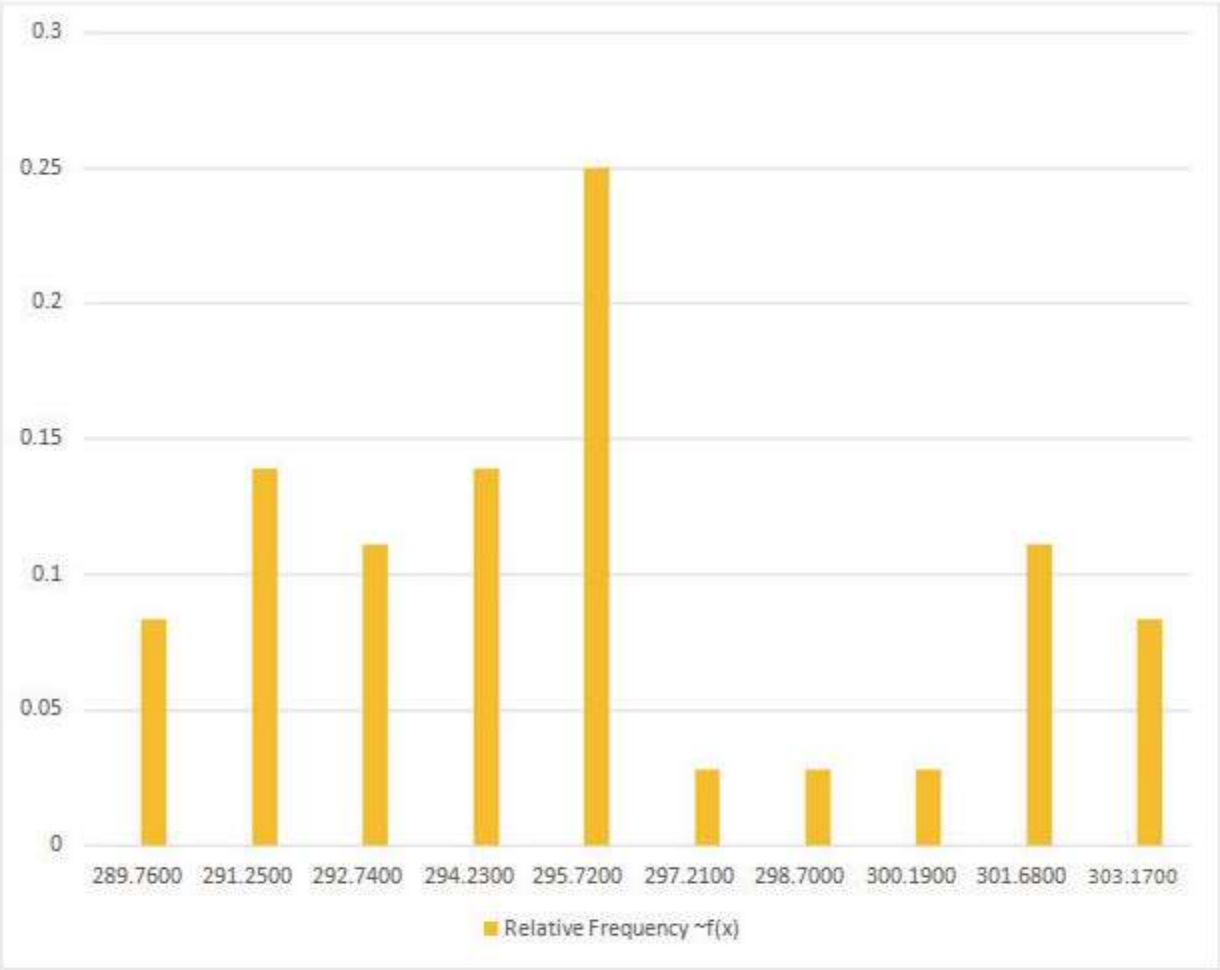


TABLE 9
STATISTICAL ANALYSIS OF UNEMPLOYMENT BENEFITS DATA

State	μ	σ	Minimum unemployment weekly	P(x>Minimum Weekly Unemployment Benefit)
Oregon	302	14.14	$538	1
Missouri	240	4.75	$320	1
Florida	232	2.01	$275	1
Arizona	214.7	4.93	$240	1
California	296.3	4.16	$450	1

DISCUSSION OF THE RESULTS AND CONCLUSIONS

The system of providing unemployment benefits by each state provides a cash cushion for employees who have been laid off. Typically, the amount of the UI is based on how much the claimant earned in wages during a base period.

A mathematical procedure was developed to provide a new method to calculate the unemployment benefits for each state. After the methodology was created, it was applied to the states California, Oregon, Missouri, Florida, Arizona, and Ohio to determine the level of UI to be distributed. From the application of this new methodology, California is liberal in providing unemployment benefits among the states that are investigated as part of this research study. Potential caveats to the development of mathematical model include the possible influence of misclassification of labor force states leading to mismeasurement of exits from unemployment and the possible misclassification of individuals as UI-eligible. The former misclassification is likely to lead to a downward bias in our estimated effects of extended benefits.

Advantages of the method proposed in this paper

Since unemployment brings with it despair, unhappiness and anguish which in turn means involves mental health, it is also a social issue. Hence the problem can be studied in the domain of social sciences without using any numbers like in this paper. In addition, the same unemployment benefits problem can be studied using rigorous mathematical techniques after the formulating the problem as a differential equation. The method used in this paper is between these two extremes and easy to understand using mathematical analysis.

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