Canadian Interest Rates and Yield Spread Dynamics

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We apply two commonly used cointegration techniques to study the relation between corporate yields and government yields and derive implications for the relation between yield spreads and government yields. Due to the stationary nature of the yield spread data, based on results of the conventional unit root tests, we cannot use cointegration theory to test this directly. The results of the unrestricted impulse response analysis provide evidence, which contradicts the results of cointegration analysis applied to corporate and government yields. Our expectation of a positive long-run relation between yield spreads and government yields is only slightly realized for AA yield spreads. The effect of a shock to the 10-year government yield appears to have a consistently negative impact on A and BBB yield spreads, both over the short-run and the long-run. The negative yield spread - government rate relation is induced due to the over-representation of callable bonds in the sample of bond indices. Moreover, yield spreads appear to exhibit characteristics similar to long-memory processes, for which the order of integration lies between zero and one. The hypothesis of fractional integration has to be tested using a completely different set of statistical tools and is not examined in this paper.

INTRODUCTION

Most models predict that the rates on corporate bonds are positively related to the corresponding rates on government bonds (Merton, 1974; Longstaff and Schwartz, 1995). However, the precise relation depends on the methodology used in deriving the model. There has been little empirical research done in this area. Extant papers focus on studying how interest rates set by the government impact corporate yield spreads.

There is a disagreement in the area of fixed-income securities with respect to the nature of the yield spread -- government rate relation. This is the focus of the current study, which is closely related to that of Duffee (1998), Jacoby, Liao, and Batten (2009), and Morris, Neal, and Rolph (2000, hereafter referred to as MNR), who investigated yield spread dynamics by estimating the effect of government rates on yield spreads not only over the long-run, but also over the short-run. The approaches of the above papers to modelling yield spread dynamics and their results differ.

Most researchers in this area apply standard regression analysis on changes over time in yield spreads as a function of changes in government rates (Duffee, 1998 and Jacoby et al, 2009). However, some (see for example, MNR) argue that this approach lacks empirical power since they find evidence that the time series of yields on corporate and government bonds are nonstationary. This may invalidate the results of the standard regression analysis. To avoid problems associated with nonstationarity, Duffee (1998) and Jacoby et al (2009) apply statistical analysis on changes in bonds' interest rates rather than their levels. However, MNR argue that by using changes in rates one may lose valuable information regarding the...
long-term relationship between yield spreads and government yields, which is reflected in the levels of the two variables. To overcome this problem one can apply cointegration analysis to test whether the variables tend to move together over the long run.

In this paper we use cointegration methodology to estimate the relation between corporate and government yields and we also derive implications for the relation between yield spreads and government yields.

We utilize Canadian corporate bond data to estimate the relationship between corporate yield spreads and government yields. Canadian bond data has several important advantages over U.S. bond data (Jacoby et al, 2009). Unlike U.S. data, Canadian bond data allows accounting for factors that may bias the estimated relationship. These factors include callability provisions commonly attached to corporate bonds, coupon and tax differential between government and corporate bonds. Thus, an analysis of the relationship between yield spreads and government bond yields using Canadian bond data should provide clean and robust results.

Longstaff and Schwartz (1995, hereafter referred to as LS) find that yield spreads are negatively related to interest rates. They account for this result by presuming that the correlation between the value of the firm's assets and the risk-free rate is negative. Their regression analysis yields a negative yield spread - treasury rate relation, decreasing in magnitude as credit rating of the bond issue increases.

Jacoby et al (2009) issues two warnings regarding to LS regression analysis. The first warning relates to the presence of callable bonds in the LS sample that might have influenced their results. He claims that by overlooking the issue of callability one may mistakenly conclude that the negative yield spread - government rate relation stems solely from the inherent default risk. The second warning concerns the LS regression model applied to relative yield spreads, a model that was shown to produce spurious results.

Duffee (1998) finds that the relation between both callable and noncallable bond yield spreads and treasury yields is negative. However, this negative relation is much stronger for callable bond issues. The relation is also found to be more negative for high-priced callable bonds than for low-priced callable bonds. This is because the call option for high-priced bonds is deeper in the money. Duffee explains the observed weak yield spread - treasury yield relation for noncallable bonds by the coupon level effect. Everything else being equal, lower coupon rates of treasury bonds as compared with corporate bonds reflect the higher duration of treasury bonds. This implies that treasury bonds will be more sensitive to changes in treasury yields and leads, he explains, to the observed negative yield spread - treasury yield relation for non-callable bonds.

Duffee (1998) also finds that the relation between the yield spreads and the slope of the treasury term structure is negative. This relation turns out to be weaker for short- and medium-maturity bonds. He then estimates the persistence of changes in the yield spreads as a result of changes in treasury yields by running unrestricted vector autoregression models (VAR) for the yield spreads, treasury yields, and the slope of the term structure. Analyzing the impulse response functions, Duffee finds that impulse error bands are quite large to produce reliable inferences. The general pattern of impulse responses indicates that it takes a long time for yield spreads and bond prices to adjust to new information.

MNR (2000) use cointegration approach to model the relation between corporate and treasury yields. They use monthly averages of daily yields for 10-year constant maturity treasury bonds and Moody's Aaa and Baa seasoned bond indices obtained from the Board of Governors of the Federal Reserve System. The sample size of each data series sums to a total of 456 observations. They show that corporate rates are cointegrated with treasury rates. Theoretically, this result suggests that the dynamics of this relationship is time-varying - the relation between corporate and treasury rates is positive in the long-run and negative in the short-run. Intuitively, this pattern implies the same time-dependent relation for the relation between yield spreads and treasury yields. To confirm this, they compute the separate impulse response functions for corporate and treasury yields as a result of a shock in the treasury yield. Then, they find the implied change in yield spreads by taking the difference between the two functions. Although innovative, this approach lacks theoretical and econometric underpinning. This is because their data for yield spreads appears to be stationary based on conventional unit root tests and, thus, unrestricted VAR impulse response function analysis is called for.
Their estimated long-run positive relation between corporate yields/yield spreads and treasury rates is consistent with models of Duan et al (1995), Lesseig and Stock (1998), and Bernanke and Gertler (1989). For example, the model of Bernanke and Gertler (1989) predicts that higher treasury rates will increase the agency problems for borrowers, widening the difference between internal and external financing costs and the firm's yield spread.

The paper is organized as follows. The second section applies both univariate and multivariate cointegration procedures to Canadian corporate and government yields. In this section we also establish the implied relation between yield spreads and government rates based on the dynamics of the corporate yield - government yield relation. Summary and conclusions are offered in the third section.

DATA, SAMPLE PROPERTIES, AND RESULTS

The sample of corporate and treasury yields is based on the month-end, yield-to-maturity indices from the Scotia Capital Markets (SCM) taken from the CANSIM database. The sample covers the December 1985 to April 2002 period. SCM provides long-term corporate indices for four different investment-grade ratings: AAA, AA, A, and BBB. Since the data for the AAA investment-grade rating is discontinued in March 1993, we exclude the AAA rating from the analysis for reason of limited sample size. SCM's long-term corporate bond indices include bonds with maturity greater or equal to 10 years. The yield spreads data is constructed as the difference in the index yield and 10-year constant maturity Government of Canada index yield. Thus, average portfolio maturity of 10 years is assumed for long-term corporate bond index. The slope of the term structure variable is proxied with the difference between the yields of the 30-year and 10-year constant maturity Government of Canada indices.

Jacoby et al (2009) argues that the SCM data are suitable to study the yield spread - government yield relation. He argues that these indices provide the flexibility to control for callability risks, coupon level effects, and effects arising from taxation.

Table 1 reports the summary statistics for corporate yields, government rates, constructed series of yield spreads and yield spread changes during the sample horizon. In general, the lower the credit rating of an index, the wider is the spread due to higher probability of default. Over the estimated period, AA yields averaged 9.54%, A yields averaged 9.78%, and BBB yields averaged 10.61%. Also, AA yields have the highest volatility among all ratings: the standard deviation of AA yields is 2.42% as compared with the standard deviation of 2.16% of the BBB yields. The AA yield spreads averaged 0.97% over the sample period, A yield spreads averaged 1.22%, and BBB yield spreads averaged 2.05%.

Table 1 also reports the results of the Jarque-Bera test of normality. Recall, if the p value of the chi-square statistic is sufficiently low, one can reject the hypothesis that the residuals are normally distributed. Looking at table 1, it is seen that all series, including corporate yields, government yields, yield spreads and the slope of the term structure, are not normally distributed. Almost all of the estimates of kurtosis statistics are less than three and this implies that probability density functions are platykurtic (thick-tailed). The estimates of kurtosis for the mean changes in corporate yields, government yields, the slope of the term structure, and BBB yield spreads show a value greater than three that implies leptokurtic (a slim or long-tailed) probability function. The normal distribution is, however, characterised by zero skewness and kurtosis of three. Thus, normal distribution is symmetric and mesokurtic. Turning to skewness measures, Table 1 shows that all corporate yields and yield spreads in levels have positive skewness. And positive skewness implies that the left tail of the distribution have more probability than a normal distribution.

We examine the quantile-quantile (QQ)-plots for corporate yields and yield spreads. These figures (available upon request) plot the quantiles of the chosen series against the normal distribution. If the two distributions are the same, the QQ-plot should lie on a 45° line. The pattern of deviation from linearity provides an indication of the nature of the mismatch; the two figures clearly indicate that yield spreads have distributions closer to the normal than corporate yields. Since all estimated plots curve downward at the left, and upward at the right, it is an indication that respective distributions are platykurtic and have a thicker tail than the normal distribution. Comparing yield spread distributions with corporate yield
distributions, it can be seen that the plots are slightly convex, indicating that distributions of yield spreads are slightly negatively skewed compared with the distributions of corporate yields.

**TABLE 1**
**SUMMARY STATISTICS FOR CORPORATE YIELDS AND YIELD SPREADS OF SCM LONG-TERM CORPORATE BOND INDICES**

This table reports the summary statistics for corporate yields, government yields, the slope of the term structure, and yield spreads. Summary statistics are reported for levels and changes in the above variables. We report the mean, the standard deviation, the skewness, the kurtosis, and the Jarque-Bera test of normality with its p-value. Level_TS is the level of the term structure represented by the 10-year Government of Canada constant maturity index. Slope_TS is the level of the term structure represented by the difference between the yields in the 30-year and 10-year Government of Canada constant maturity indices.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AA</td>
<td>239</td>
<td>9.5400</td>
<td>2.4300</td>
<td>0.3300</td>
<td>2.7000</td>
<td>5.14</td>
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<td>A</td>
<td>239</td>
<td>9.7800</td>
<td>2.3900</td>
<td>0.3500</td>
<td>2.8500</td>
<td>5.15</td>
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<tr>
<td>BBB</td>
<td>239</td>
<td>10.6100</td>
<td>2.1600</td>
<td>0.1100</td>
<td>2.8900</td>
<td>0.62</td>
</tr>
<tr>
<td>Level_TS</td>
<td>239</td>
<td>8.5600</td>
<td>2.3900</td>
<td>0.3700</td>
<td>2.6600</td>
<td>6.67</td>
</tr>
<tr>
<td>Slope_TS</td>
<td>239</td>
<td>0.3000</td>
<td>0.2700</td>
<td>-0.2400</td>
<td>2.6700</td>
<td>3.36</td>
</tr>
<tr>
<td>ΔAA</td>
<td>238</td>
<td>-0.0500</td>
<td>0.3500</td>
<td>-0.3700</td>
<td>4.2800</td>
<td>21.78</td>
</tr>
<tr>
<td>ΔA</td>
<td>238</td>
<td>-0.0500</td>
<td>0.3400</td>
<td>-0.5200</td>
<td>4.4700</td>
<td>31.88</td>
</tr>
<tr>
<td>ΔBBB</td>
<td>238</td>
<td>-0.0400</td>
<td>0.4100</td>
<td>0.2900</td>
<td>6.3800</td>
<td>116.61</td>
</tr>
<tr>
<td>ΔLevelTS</td>
<td>238</td>
<td>-0.0400</td>
<td>0.3700</td>
<td>-0.0600</td>
<td>4.3400</td>
<td>17.86</td>
</tr>
<tr>
<td>ΔSlopeTS</td>
<td>238</td>
<td>0.0010</td>
<td>0.1100</td>
<td>-0.2100</td>
<td>3.3000</td>
<td>2.69</td>
</tr>
<tr>
<td>AA_CS</td>
<td>239</td>
<td>0.9800</td>
<td>0.2800</td>
<td>0.5300</td>
<td>2.9800</td>
<td>11.00</td>
</tr>
<tr>
<td>A_CS</td>
<td>239</td>
<td>1.2200</td>
<td>0.3300</td>
<td>0.3600</td>
<td>2.4800</td>
<td>7.80</td>
</tr>
<tr>
<td>BBB_CS</td>
<td>239</td>
<td>2.0500</td>
<td>0.8300</td>
<td>1.3800</td>
<td>5.2900</td>
<td>127.92</td>
</tr>
<tr>
<td>ΔAACS</td>
<td>238</td>
<td>-0.0010</td>
<td>0.1800</td>
<td>0.2300</td>
<td>4.7700</td>
<td>33.09</td>
</tr>
<tr>
<td>ΔACS</td>
<td>238</td>
<td>-0.0020</td>
<td>0.1800</td>
<td>0.0800</td>
<td>4.4100</td>
<td>20.03</td>
</tr>
<tr>
<td>ΔBBBCS</td>
<td>238</td>
<td>0.0030</td>
<td>0.3100</td>
<td>1.7300</td>
<td>20.0700</td>
<td>3007.30</td>
</tr>
</tbody>
</table>

The statistics are based on a monthly SCM corporate bond indices data from 1985:12 to 2002:4.

We now summarize the dynamics of the corporate yield - government yield relation and derives some implications with respect to the yield spread - government yield relation. The estimation and inference will be drawn from unit root testing, cointegration, and error-correction estimation. We look at both univariate and multivariate testing procedures and compare the performance of univariate residual-based tests of Engle-Granger to that of Johansen maximum likelihood system-based tests. The existence of a bivariate cointegrating vector between corporate yields and government yields is analyzed. This section provides evidence that corporate yields and the level of the term structure are non-stationary and thus can indeed be on the same wavelength. Yield spreads appear to be stationary, meaning that standard cointegration theory framework cannot be applied to test the yield spread - government yield relation directly.

Impulse response functions to the shocks in risk-free rate can provide a useful and valid way to analyze the short- and long-run dynamics of these relations. A standard proposition in the related literature is that real disturbances (shocks in government rates) may cause permanent (long-run) and/or temporary short-run deviations of actual yield spreads from equilibrium spreads. The data show that
impulse response analysis of the corporate yield - government yield relation has to be estimated by restricted VAR analysis. However, due to the small sample size of the estimated sample, as well as the drawbacks of cointegration testing procedures themselves, impulse response functions in this context were not attempted in this study. Moreover, the yield spread data appear to be stationary and unrestricted VAR analysis is called for to find out the response of yield spreads to a one standard deviation innovation in the 10-year government rate.

The primary objective of the following discussion is to apply cointegration testing procedures to corporate and government yields of the SCM sample and to make intuitive inferences about the impact of government yields on corporate yield spreads.

**Corporate Yield - Government Yield Relation**

We find autocorrelations for corporate and government yields, and the slope of the term structure. For all of the above-mentioned variables, autocorrelation coefficients are quite large up to a lag 6, meaning that the series under consideration exhibit some persistence.

Augmented Dickey-Fuller and Phillips-Perron tests that have unit root as the null hypothesis investigate the presence of a random walk component in each series. To save space, we do not report the unit root test results but they are available upon request. The null hypothesis of a unit root was accepted for all series using both the augmented Dickey-Fuller tests and the Phillips-Perron tests. For the yield spreads, the results of the unit root tests are different. The Dickey-Fuller test results show that yield spreads contain unit root. But using the Phillips-Perron test, we are able to reject the null hypothesis of a unit root at all lags for yield spreads. Applying the Kwiatkowski et al (1992) test, we are also unable to reject the hypothesis of level stationarity and trend stationarity for AA and A yield spread data series at the 5% level and for BBB yield spread data at the 1% level. This confirms the preliminary evidence that yield spreads are stationary. The monthly changes in corporate yields, the level, and the slope uniformly exhibit stationarity. This means that corporate yields, the level, and the slope of the term structure are integrated of order 1, whereas the yield spread data appears to be stationary.

Table 2 presents the results of Engle-Granger equilibrium regressions for three corporate yields: AA, A, BBB. Table 3 reports the results of unit root testing procedures applied to residuals from the equilibrium regressions. Since the estimated residuals from the equilibrium regressions have a zero mean and do not have a time trend, the tests include only lagged residuals and their differenced lagged values as regressors. An iterative procedure is followed to determine the appropriate lag length. 12 lagged difference terms are added to the regression equation. Then the lag length is determined by minimizing the Schwartz criterion for augmented Dickey-Fuller tests and by selecting the bandwidth parameter $\ell$ for the kernel-based estimators of Johansen system-based tests. The bandwidth selection is based on the Newey-West procedure. Comparing the estimated $\tau$-values with calculated critical values for the null of no cointegration from J.G. MacKinnon (1993), it can be seen that in absolute terms the estimated $\tau$-values exceed the critical values at least at the 5% level of significance. Thus, there is sufficient evidence that corporate and government yields are cointegrated based on the Engle-Granger cointegration methodology. The Engle-Granger approach indicates that the relation between corporate and government yields is time-varying: positive in the long-run and negative in the short-run.
TABLE 2
ENGLE-GRANGER EQUILIBRIUM REGRESSIONS FOR CORPORATE AND GOVERNMENT YIELDS

This table reports the results of the first step of the Engle-Granger cointegration procedure. We report the results of a simple OLS regression analysis for corporate and government yields.

<table>
<thead>
<tr>
<th>Variable</th>
<th>AA</th>
<th>Coefficient</th>
<th>t-statistic</th>
<th>A</th>
<th>Coefficient</th>
<th>t-statistic</th>
<th>BBB</th>
<th>Coefficient</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.8858</td>
<td>13.1068</td>
<td></td>
<td>1.2781</td>
<td>16.2038</td>
<td></td>
<td>3.3254</td>
<td>18.4097</td>
<td></td>
</tr>
<tr>
<td>Level_TS</td>
<td>1.0107</td>
<td>132.917</td>
<td></td>
<td>0.993</td>
<td>111.8946</td>
<td></td>
<td>0.8512</td>
<td>41.8811</td>
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<tr>
<td>R²</td>
<td>0.9868</td>
<td></td>
<td></td>
<td>0.9814</td>
<td></td>
<td></td>
<td>0.881</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DW</td>
<td>0.4246</td>
<td></td>
<td></td>
<td>0.3017</td>
<td></td>
<td></td>
<td>0.1614</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE 3
ENGLE-GRANGER TESTS FOR COINTEGRATION BETWEEN CORPORATE AND GOVERNMENT YIELDS

The residuals from each equilibrium regression were checked for unit roots. The unit root tests in a cointegration context are computationally easy to do. Since the estimated residuals from the equilibrium regressions have a zero mean and do not have a time trend, the tests included only lagged residuals and their differenced lagged values as regressors. An iterative procedure was followed to determine the appropriate lag length. 12 lagged difference terms were added to a regression equation. Then the lag length was determined by minimizing the Schwartz criterion for the augmented Dickey-Fuller tests and by selecting the bandwidth parameter ℓ for the kernel-based estimators of Ω₀. The bandwidth selection was based on the Newey-West procedure. **, *, and *** means that the variable under consideration is stationary at the 10% level, 5%, and 1% level respectively.

<table>
<thead>
<tr>
<th>Indices</th>
<th>AA</th>
<th>Coefficient</th>
<th>t-statistic</th>
<th>A</th>
<th>Coefficient</th>
<th>t-statistic</th>
<th>BBB</th>
<th>Coefficient</th>
<th>t-statistic</th>
</tr>
</thead>
</table>

Table 4 presents the results of Johansen maximum likelihood approach to test for the existence of a bivariate cointegrating vector between corporate and government yields. These results are based on the appropriate lag length determined by minimizing the Schwartz Criterion. We find the evidence of 2 cointegrating vectors for AA rates, and no cointegrating vectors for A and BBB rates at the 5% level using this approach. At the 1% significance level, however, we find no cointegrating vectors for all three rates. Consider, for example, the results for the A-rated index. For this index, the first eigenvalue statistic is not significant at the 5% level. Therefore, we are unable to reject the hypothesis of zero cointegrating vectors in favour of the alternative that there exists one cointegrating vector. The second maximum eigenvalue statistic is significant and supports the existence of two cointegrating vectors. However, since the existence of one cointegrating relation is rejected, the existence of two cointegrating relations can be eliminated. The results for other series are very similar.
TABLE 4
MULTIVARIATE COINTEGRATION RESULTS FOR CORPORATE AND GOVERNMENT YIELDS

This table presents the results of the Johansen multivariate test. The statistics are based on a monthly long-term SCM corporate bond indices data from 1985:12 to 2002:4. Level_Ts is the level of the term structure represented by the 10-year Government of Canada constant maturity index. Slope_Ts is the slope of the term structure represented by the difference between the yields in the 30-year and 10-year Government of Canada constant maturity indices.

<table>
<thead>
<tr>
<th>Cointeg.Group</th>
<th>Lags</th>
<th>Maximal Eigenvalue Statistic</th>
<th>Trace Statistic</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Eigenvalues</td>
<td>Statistic</td>
</tr>
<tr>
<td>AA, Level</td>
<td>1</td>
<td>0.0712</td>
<td>17.5035*</td>
</tr>
<tr>
<td></td>
<td>λ</td>
<td>0.0377</td>
<td>9.0999*</td>
</tr>
<tr>
<td>A, Level</td>
<td>1</td>
<td>0.0572</td>
<td>13.9492</td>
</tr>
<tr>
<td></td>
<td>λ</td>
<td>0.9368</td>
<td>8.8998*</td>
</tr>
<tr>
<td>BBB, Level</td>
<td>1</td>
<td>0.0416</td>
<td>10.0679</td>
</tr>
<tr>
<td></td>
<td>λ</td>
<td>0.4554</td>
<td>7.0764*</td>
</tr>
</tbody>
</table>

If the two variables are cointegrated, then they are on the same wavelength, or the trends in corporate and government yields cancel out. This would have a twofold meaning. First, if we plot the cointegrating relations between corporate yields and the 10-year Government of Canada yield, it should not on average significantly deviate away from a zero line. Examining cointegrating relations between corporate yields and the Government of Canada yield (see Figure 1), it can be seen that all of these lines are close on average to the zero line. Thus, it can be concluded that our corporate yields and government rates are cointegrated, a result that implies that an Engle-Granger test procedure has more power than the Johansen test procedure. This is in line with theoretical research in this area. Second, although the Johansen test shows no cointegration for corporate and government yields, the estimates of the long-run relation between corporate and government rates based on Engle-Granger and Johansen test procedures are approximately equal in magnitude. For instance, the cointegrating vector for AA rate using the Engle-Granger methodology is (1, -1.0107) vs (1, -1.0258) using the Johansen approach.

FIGURE 1
COINTEGRATING RELATIONS BETWEEN CORPORATE AND GOVERNMENT YIELDS

These graphs sketch the stationary linear combinations of corporate and government yields or, in other words, cointegrating equations. Cointegrating relations sketched below represent the long-run equilibrium relationship among the variables, which should not significantly deviate from a zero line. The data covers the period from December 1985 to April 2002.
The results of the more powerful Engle-Granger approach indicate that corporate yields and government yields are cointegrated. Since a 1% increase in the government yield generates an increase in corporate yields, it can be claimed that as interest rates rise, yield spreads will eventually widen as well.

The results of the Johansen maximum likelihood approach are puzzling, taking into consideration that this test is biased towards the acceptance of the alternative hypothesis of cointegration. But we argue that the Engle-Granger approach is better suited for the purpose of testing the corporate yield - government yield relation in our sample. This is because our corporate and government yield series are not normally distributed and only one lag was included in the estimation. The Johansen tests are inferior to the Engle-Granger tests when the data deviate from the assumption of normally, identically and independently distributed disturbances and when the lag length/sample size is small.

An alternative explanation for the non-existence of a bivariate cointegrating vector between corporate and government yields based on the Johansen test results and for the weak existence of this vector based on the Engle-Granger test results is as follows. Plotting the autocorrelation function for the error-correction term of the equilibrium regression for the AA rate in the Figure 2 below, it can be observed that although autocorrelation coefficients appear to be stationary, they exhibit some long-memory attributes. Autocorrelation coefficients do not exhibit the same type of persistence as they do for the original series.

**FIGURE 2**
**CORRELOGRAM FOR THE AA YIELD ERROR-CORRECTION TERM**

This figure plots the 200 sample autocorrelation coefficients for the estimated error-correction term of the equilibrium regression involving the AA rate. The data covers the period from December 1985 to April 2002.

Thus, the error-correction term decays more slowly to zero as a result of a shock than the usual exponential decay of the autocorrelation function for the covariance stationary and invertible autoregressive moving average process. This means that the effect of a shock is long lasting and that the error-correction term might be following a fractionally integrated process of the form:

\[(1-L)^d (e_t - \mu) = u_t, \text{ where } u_t \text{ is integrated of order 0.}\]

Several researchers have looked at the power of cointegration tests in the presence of fractionally integrated series and concluded that the Johansen test has very little power against fractional alternatives (see, for example, Gonzalo (1998); Cheung and Lai (1993)). Thus, we argue that the Engle-Granger procedure is more robust to the misspecification of the long-memory components of variables entering the model as well as to the fractionally integrated deviations from the long-run equilibrium.
Based on the results of the Engle-Granger tests, we estimate the corporate yield-government yield error-correction models. We use the residuals obtained from the equilibrium regressions as an instrument for the error-correction terms, as proposed by Engle and Granger. Other than the error-correction term, obtained from the equilibrium regression, the error-correction model constitutes VAR in first differences and will be applied in the VAR framework. Thus, OLS is an efficient estimation strategy since each equation contains the same set of regressors. Moreover, since all terms in the error-correction model are stationary, the test statistics used in traditional VAR analysis can also be used here. The lag length for our error-correction models was gauged by minimizing the Schwartz criterion.

Table 5 reports the results of the error-correction models. The point estimates of the error-correction terms in the model are all insignificantly different from zero. This might indicate that movements in corporate yields are independent of movements in government yields. Moreover, the point estimate for the AA rate error-correction term is positive, casting doubt on the existence of cointegration between AA corporate yield-government yield relation altogether.

**TABLE 5**

**ESTIMATES OF THE ERROR-CORRECTION MODEL FOR CORPORATE AND GOVERNMENT YIELDS**

This table presents the results of the error-correction bivariate tests, which are based on the Engle-Granger univariate method. The following error-correction regression was estimated.

<table>
<thead>
<tr>
<th>Variable</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>t-statistic</td>
<td>Coefficient</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.0371</td>
<td>(-1.6535)</td>
<td>-0.0367</td>
</tr>
<tr>
<td>ΔAA_{t-1}</td>
<td>0.1212</td>
<td>-0.8626</td>
<td>0.1652</td>
</tr>
<tr>
<td>ΔA_{t-1}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔBBB_{t-1}</td>
<td>0.0432</td>
<td>-0.3308</td>
<td>0.0012</td>
</tr>
<tr>
<td>ΔLevelTS_{t-1}</td>
<td>0.0223</td>
<td>-0.2642</td>
<td>-0.0232</td>
</tr>
<tr>
<td>ECterm</td>
<td>0.0275</td>
<td>0.0282</td>
<td>0.021</td>
</tr>
<tr>
<td>R²</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Impulse response function analysis is not attempted here due to the expected biased nature of cointegration relationship that arises when the variables entering the system are not normally distributed and the sample size is small. The small power of cointegration tests against the fractional alternatives leads us to believe that the cumulative impulse response corresponding to a shock in the infinite past will be zero for the processes integrated of order less than one.

Phyllips (1998) shows that the estimated impulse responses in a cointegrated VAR model when based on the reduced rank regression are consistent if the cointegrating rank is consistently estimated. Since we feel that the cointegrating rank is underestimated, the impulse response function analysis is not appropriate in this setting. Moreover, Kilian (1998) stresses that the small sample distribution of impulse responses can be significantly biased and skewed. He shows that when this is true, it can make traditional confidence intervals extremely inaccurate.
Yield Spread - Government Yield Relation

Since yield spreads are integrated of order zero, it is inappropriate to estimate the yield spread - government yield relation directly by applying standard cointegration theory.

Considering the distributional patterns of yield spreads, it can be seen from the Jarque-Bera normality test results and QQ-plots that yield spreads deviate from the normal distribution. Although this deviation is small (the skewness and kurtosis statistics are close to zero and three respectively), the kurtosis statistics are all less than three meaning that probability distribution functions are fat-tailed. Pedrosa and Roll (1986) document similar results for yield spreads and their changes. Heavier tails compared to the standard normal density mean that there is a higher probability of extreme observations. Given the later finding and knowing that critical values of the unit root tests and cointegration tests are biased in the presence of non-normal, non-independent, and non-identical distribution of disturbances, we cannot use the standard cointegration theory to test the yield spread - government yield relation.

The direct analysis of the yield spread - government yield relation is more reasonable from a theoretical perspective. Researchers are interested in how the yield spread responds to the shifts in the government yield. Some researchers apply regression analysis to yield spread changes instead of levels (see for example, Duffee, 1998 and Jacoby et al, 2009). However, the theory discusses the relation between the levels of these variables and not their changes. Therefore, it is important to apply unit root testing procedures to the levels of yield spreads and government yields. If the variables are non-stationary then the researcher should not difference the non-stationary series to apply conventional regression analysis, since important and valuable information can be lost.

Figures 3 and 4 demonstrate that the relation between changes in spreads and changes in the government yield is different from the relation between the levels of these variables. The relation between changes of yield spreads and government yields seems to be negative for AA, and A indices and there seems to be no relation between these variables for the BBB index. The relation between the AA yield spread in level form and the government yield in the level form appears to be positive, whereas that for the A and BBB indices appears to be negative.

FIGURE 3
CHANGES IN YIELD SPREADS VERSUS CHANGES IN GOVERNMENT RATES (LONG-TERM INDICES)

These figures plot changes in yield spreads as a function of changes in government yields. The data covers the period from December 1985 to April 2002.
These figures plot yield spreads as a function of government yields. The data covers the period from December 1985 to April 2002.

Since the unit root results suggest that yield spreads are stationary, whereas the level of the term structure is non-stationary, cointegration theory cannot be used to understand the nature of the relationship between yield spreads and government yields. However, unrestricted VAR analysis can and should be applied to estimate impulse responses of yield spreads to a one standard deviation innovation in a 10-year government rate. The ordering of the variables is: 10-year government yield, the appropriate yield spread. Figure 5 plots impulse responses of yield spreads as a result of a one standard deviation innovation in a government rate. First, the standard error bands are wider for BBB-rated bonds than those for A-, and AA-rated bonds. This means that reliable inferences can be made, especially at shorter horizons. The relation between the A, BBB yield spreads and the government rate seems to be consistently negative at long horizons, slowly pulling towards zero as time progresses. Consistent with the AA corporate yield - government yield estimated relation, the AA yield spread falls initially by 0.06 standard deviations, but ultimately increases by 0.0004 standard deviations by month 16 as a result of a shock in the 10-year government yield. Consistent with the findings of Duffee (1998), the impulse response results show that the responses of yield spreads to shocks in government yields take a long time to dissipate.

The results of the unrestricted impulse response analysis are puzzling. The long-run effect of a shock to the 10-year government yield appears to have a positive long-run impact on the AA yield spreads but negative long-run impact on the A and BBB yield spreads. An explanation can be provided by Jacoby et al (2009), who claims that most Canadian corporate bonds issued starting from the year of 1987 carry the "doomsday" call provision. This call provision provides means to control for the callability bias. Specifically, he asserts that starting from the year 1995, vast majority of bonds in the SCM long-term indices category carry a doomsday call provision instead of a standard call provision. Recalling that it will typically be economically suboptimal for firms to call BBB-rated bonds carrying a doomsday call provision, he argues that the BBB index starting from 1995 can be assumed to be economically non-callable. However, since the sample period from 1985 to 1995 is dominated by bond issues carrying a standard call provision, the callability bias is applicable to our estimation period. Duffee (1998) argues that the presence of callable bonds in the sample makes the relation between yield spreads and government yields negative. This implies that the cumulative impact of callability in our sample is larger for the A and BBB yield spreads rather than for the AA yield spreads.
FIGURE 5
IMPULSE RESPONSES OF YIELD SPREADS TO A SHOCK IN THE 10-YEAR GOVERNMENT YIELD (LONG-TERM INDICES)

Each graph represents the impulse response of long-term AA, A and BBB yield spreads to the shock in the 10-year constant maturity Government of Canada yield implied by a vector autoregression with two lags of 10-year constant maturity Government of Canada yields, and the given yield spread, in that order. The standard error bounds on the impulse responses are also displayed. The data covers the period from December 1985 to April 2002.

Since it is known that unit root tests have very small power to detect fractional integration root parameters, we have hypothesized that yield spread data is fractionally cointegrated. Empirically, if the root differs from one in either direction, then a different set of statistical tools is called for. This drawback of the unit root tests is also shared by the cointegration testing procedures, implying that conventional tests are unable to capture the existence of possible fractional cointegration between yield spreads and government yields. Next, we examine the autocorrelation functions of yield spreads by plotting the autocorrelation coefficients against 200 lag parameters for the AA yield spread data in Figure 6. The

FIGURE 6
CORRELOGRAM FOR THE AA YIELD SPREAD

The figure plots the first 200 sample autocorrelation coefficients for the AA yield spread. The monthly long-term SCM yield spread data covers the period from December 1985 to April 2002.
following pattern emerges: the autocorrelation coefficient function appears to be generated by a stationary
process, but this process seems to exhibit long memory characteristics. Thus, yield spreads appear to: (i)
possess non-normal distribution with fat tails; (ii) have autocorrelations that decay to zero very slowly;
and (iii) have cycles, which are not periodic. This might indicate that yield spreads have a fractional order
of integration.

CONCLUSION

This paper applies two commonly used cointegration techniques to study the relation between
corporate yields and government yields. This sign and significance of this relation has important
implications for the relation between yield spreads and government yields.

The hypothesis of non-stationarity for corporate and government yields is not rejected. However, first
differences of these time series are uniformly stationary. This has an important implication for academic
circles, namely that one should be extremely precautious to use corporate yields in the level form in
statistical applications. One can only do that in the case if corporate yields and the other series under
consideration are cointegrated.

We find preliminary evidence of the existence of cointegrating relation between corporate and
government yields based on the Engle-Granger method. The Engle-Granger residual-based tests provide
evidence that corporate yields and the 10-year Government of Canada yield are cointegrated, implying
that there is a long-run equilibrium relationship between these variables. Unfortunately, error-correction
estimates are all insignificantly different from zero. This might indicate that movements in corporate
yields are independent of movements in government yields. Moreover, the point estimate for the AA rate
error-correction term is positive, casting doubt on the existence of cointegration between AA corporate
yield and the 10-year government yield altogether.

Consistent with the findings of Duffee (1998), our results also show that the responses of yield
spreads as a result of shocks in the 10-year government rate take a long time to dissipate. This can be
partly explained by the possible long memory characteristics exhibited by the yield spread series, for
which the order of integration lies between 0 and 1. Estimates of the exact order of integration of yield
spreads should be of interest to policy makers for at least two reasons. First, this will enable them to
determine whether shocks to yield spreads are short-lived, long-lived or infinitely lived. Second, if the
order of integration is less than one, then we can suspect that the cointegrating relation involving yield
spreads may not be precisely of zero order, with the consequence that adjustments to re-establish the
long-run equilibrium state may follow long-memory processes as it appears to be. In case of yield
spreads, significant shocks to interest rate expectations, such as those resulting from changes in policy
regimes, may indeed take a long time to dissipate. This implies that it will take a long time for yield
spreads to revert to their respective means. Thus, forecast accuracy might be improved only within the
framework of longer-term forecasts. Even though estimates of the exact order of integration are extremely
important, they have to be calculated using a completely different set of statistical tools and are not
looked at in this paper.

REFERENCES


